Statistics
Explanation of Population Proportion Confidence Interval (CI) Formula

Example: What proportion of ESU students drink coffee? We sampled 155 ESU students, and 82 of them told us they drink coffee.

Population = all ESU students	Sample = 155 ESU students who were asked ($n = 155$)
Parameter = p = proportion of all ESU students who drink coffee (unknown)	Statistic = $\hat{p}$ = proportion of the sample who drink coffee

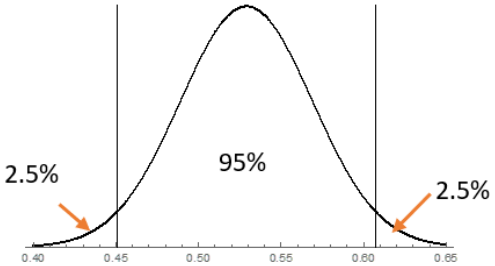
$$\hat{p} = \frac{82}{155} = 0.529$$

From Chapter 6: The Distribution of $\hat{p}$ is approximately normal

	Actual	Approximated
Mean	p	$\hat{p} = 0.529$
SD	$\sqrt{\frac{p(1-p)}{n}}$	$\text{SE} = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = \sqrt{\frac{0.529(1-0.529)}{155}} = 0.04$

Idea Behind the CI: The unknown parameter p is in the center of the distribution. By using the unbiased estimator $\hat{p}$ as the center and approximating the middle 95% of the distribution, we have a 95% chance that we found p.

We get a 95% Confidence Interval for p by finding the middle 95% of this distribution

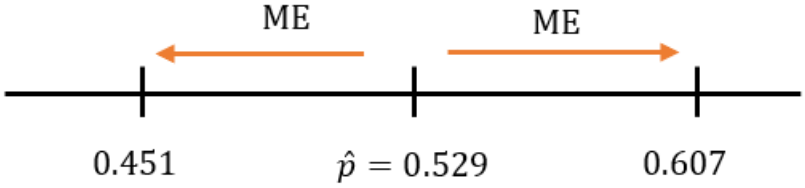


$\text{invNorm}(0.025, 0.529, 0.04) = 0.451$

$\text{invNorm}(0.975, 0.529, 0.04) = 0.607$

- A 95% CI for the proportion of ESU students who drink coffee is (0.451, 0.607)
- Correct Interpretation: There's a 95% chance that the interval (0.451, 0.607) contains the proportion of ESU students who drink coffee.

Picture of the Confidence Interval

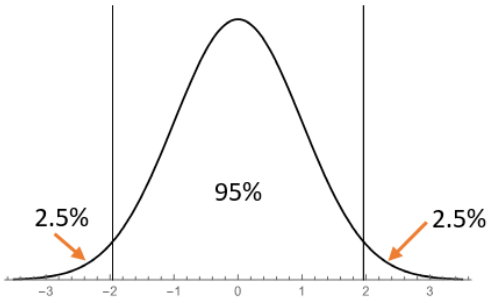


PSchembari

- The Margin of Error (ME) is 0.607 – 0.529 = 0.078 (you can find this in a few different ways)
- For the CI, we start at $\hat{p}$ in the center and then add and subtract ME.
- Another way to write the CI is

$$\hat{p} \pm ME = 0.529 \pm 0.078$$

The Margin of Error formula is based on the Standard Normal Curve z (Mean 0, SD 1). Here is the middle 95%:



invNorm(0.025, 0, 1) = –1.96

invNorm(0.975, 0, 1) = 1.96

The z-score 1.96 is called a **critical value**

In the example, we had ME = 0.078, SE = 0.04

Notice that ME = 0.078 = 1.96 * 0.04 = 1.96 * SE

Formula for the Margin of Error

$$ME = z^*(SE) = z^*\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

We put it all together to get:

Formula for the Confidence Interval for p

$$\text{Point Estimate} \pm \text{ME} = \hat{p} \pm z^*\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$