

SETS – Equal, Equivalence, and Subsets

Two sets A and B are **equal** if they have exactly the same elements. Every element of A must be an element of B and every element of B must be an element of A . The order the elements are written does not matter.

We write $A = B$.

Example: $A = \{x, y, z, e, f\}$ is equal to the set $B = \{e, x, f, y, z\}$ but A is not equal to the set $C = \{x, y, z, e, f, g\}$

Sets A and B are **equivalent** if $n(A) = n(B)$, they have the same number of elements. The sets can be put in a **one-to-one correspondence**.

Example: $A = \{1, 2, 3\}$ and $B = \{4, 5, 6\}$

$A \neq B$ but A is **equivalent** to B . Both sets have 3 elements. We can “match up” every element of A with an element of B and vice versa (one-to-one correspondence)

The set A is a **subset** of the set B if every element of A is also an element of B . We write $A \subseteq B$. If there is an element of A that is not in B then A is not a subset of B .

Example: For the following sets $P = \{5, 10, 15, 20, 25, 30\}$ and $L = \{10, 20, 30\}$ and $X = \{25, 30, 35\}$
 L is a subset of P but X is not a subset of P .

Some properties of subsets:

- 1) Every set is a subset of itself: $P \subseteq P$
- 2) The empty set is a subset of every set: $\emptyset \subseteq P$

The set A is a **proper subset** of the set B if A is a subset of B but $A \neq B$. That means there is at least 1 element in B that is not in A . We write $A \subset B$

Example: The set L in the previous example is a proper subset of P .

Number of subsets of a given set: A set that has k elements has 2^k subsets (including the empty set and the entire set itself) and $2^k - 1$ proper subsets (because we exclude the entire set).

Example: List the subsets of the set $F = \{\text{red, white, blue}\}$

Subsets:

$\emptyset$
 $\{\text{red}\}$ $\{\text{white}\}$ $\{\text{blue}\}$
 $\{\text{red, white}\}$ $\{\text{red, blue}\}$ $\{\text{white, blue}\}$
 $\{\text{red, white, blue}\}$

The set F has 3 elements and $2^3 = 8$ subsets and $2^3 - 1 = 7$ proper subsets.