SETS – Equal, Equivalence, and Subsets

Two sets A and B are **equal** if they have *exactly the same* elements. Every element of A must be an element of B and every element of B must be an element of A. The order the elements are written does not matter.
We write $A = B$.
Example: $A = \{x, y, z, e, f\}$ is equal to the set $B = \{e, x, f, y, z\}$ but A is not equal to the set $C = \{x, y, z, e, f, g\}$

Sets A and B are ***equivalent*** if $n(A) = n(B)$, they have the same number of elements. The sets can be put in a **one-to-one correspondence.**
Example: $A=\{1, 2, 3\}$ and $B=\{4, 5, 6\}$
$A \neq B$ but A is equivalent to B. Both sets have 3 elements. We can "match up" every element of A with an element of B and vice versa (one-to-one correspondence)

The set A is a **subset** of the set B if every element of A is also an element of B. We write $A \subseteq B$. If there is an element of A that is not in B then A is not a subset of B.

Example: For the following sets $P = \{5, 10, 15, 20, 25, 30\}$ and $L = \{10, 20, 30\}$ and $X = \{25, 30, 35\}$
L is a subset of P but X is not a subset of P.

Some properties of subsets:
1) Every set is a subset of itself: $P \subseteq P$
2) The empty set is a subset of every set: $\emptyset \subseteq P$

The set A is a **proper subset** of the set B if A is a subset of B but $A \neq B$. That means there is at least 1 element in B that is not in A. We write $A \subset B$
Example: The set L in the previous example is a proper subset of P.

Number of subsets of a given set: A set that has k elements has 2^k subsets (including the empty set and the entire set itself) and 2^k-1 proper subsets (because we exclude the entire set).
Example: List the subsets of the set F = {red, white, blue}
Subsets:
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{red} {white} {blue}
{red, white} {red, blue} {white, blue}
{red, white, blue}

The set F has 3 elements and 2^3=8 subsets and 2^3-1 = 7 proper subsets.