Place Value Numeration Systems

Base-10 (Hindu-Arabic) uses 10 single digits (0, 1, 2, 3, 4, 5, 6, 7, 8, 9) to form a number.

It is a **place value system**, which means the position a digit is in has a place value based on the powers of 10.
For example, 200 is 2 hundreds (10^2) but 20 is 2 tens (10^1) and 2 is 2 ones (10^0).

Place value name	Million	Hundred-thousand	Ten-thousand	Thousand	Hundred	Ten	Ones
Power of ten	10^6	10^5	10^4	10^3	10^2	10^1	$10^0=1$

We can write a number in **expanded notation** by writing the face value times the place value.
For example: 1,234,567 in expanded form is $1*10^6 + 2*10^5 + 3*10^4 + 4*10^3 + 5*10^2 + 6*10^1 + 7*1$

Counting: When we count in base-10 we are adding 1 to the ones place value.
Every time there is a "group" of 10 in any place value it is carried over to the next place value.

0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 (ten = 1 group of 10 and 0 ones)
11, 12, 13, 14, 15, 16, 17, 18, 19, 20 (19 = 1 group of 10 & 9 ones, 20 = 2 groups of 10 and 0 ones),…,
91, 92, 93, 94 95 96 97 98, 99, 100 (93 = 9 groups of 10 & 3 ones, 100 = 1 group of 100 and 0 tens and 0 ones)

For bases other than 10, a subscript at the end of the number indicates the base.

Base-5 uses only 5 digits (0, 1, 2, 3, 4) to form a number.

The **place value** of a digit is based on **powers of 5**. The place value powers of 5 increase as we "read" the numeral from *right* to *left*.
For example, the base-5 numeral 12304_5 in expanded form is $1*5^4 + 2*5^3 + 3*5^2 + 0*5^1 + 4*1$

Counting: When we count in base-5 we are adding 1 to the digit in the ones place value.
Every time there is a "group" of 5 in a place value it is carried over to the next larger place value.

0_5, 1_5 , 2_5 , 3_5 , 4_5 ,10_5 , (10_5 = 1 group of 5 and 0 ones)
11_5 , 12_5 , 13_5 , 14_5 , 20_5 ,
(11_5 = 1 group of 5 & 1 one, 12_5 = 1 group of 5 & 2 ones, 13_5 = 1 group of 5 & 3 ones,
14_5 = 1 group of 5 and 4 ones and 20_5 = 2 groups of 5 and zero ones), …,
41_5 , 42_5 , 43_5 , 44_5 , 100_5 , (100_5 = 1 group of 5^2, zero 5s and 0 ones), …

To convert a base-5 number to base-10: multiply each face value by its place value.
For example, the base-5 numeral 12304_5 in base-10 is
$1*5^4 + 2*5^3 + 3*5^2 + 0*5^1 + 4*1 = 1*625 + 2*125 + 3*25 + 0*5 + 4*1 = 954$

To convert a base-10 to base-5: we need to divide by powers of 5. Start with the largest possible power of 5 that is less than the base-10 number.

For example, write 2146 in base-5

Powers of 5	5^5=3125	5^4=625	5^3=125	5^2=25	5^1=5	5^0=1
	bigger	2146 ÷ 625	271 ÷ 125	21÷ 25	21 ÷ 5	1 ÷ 1
quotient	than 2146	***3***	***2***	***0***	***4***	***1***
remainder		271	21	21	1	0

The equivalent base-5 number is the quotients: $\mathbf{32041_5}$

The above processes can be used in any number base.
Remember that the place values need to represent the number base being used

Base-3 uses only 3 digits (0, 1, 2) to form a number.
Place values from right to left are 1, 3^1, 3^2, 3^3, 3^4, …

Base-4 uses only 4 digits (0, 1, 2, 3) to form a number.
Place values from right to left are 1, 4^1, 4^2, 4^3, 4^4, …

Base-6 uses only 6 digits (0, 1, 2, 3, 4, 5) to form a number.
Place values from right to left are 1, 6^1, 6^2, 6^3, 6^4, …

Base-8 uses only 8 digits (0, 1, 2, 3, 4, 5, 6, 7) to form a number.
Place values from right to left are 1, 8^1, 8^2, 8^3, 8^4, …

Base-9 uses only 9 digits (0, 1, 2, 3, 4, 5, 6, 7, 8) to form a number.
Place values from right to left are 1, 9^1, 9^2, 9^3, 9^4, …

For **bases larger than 10** we need more than 10 *single* digits to form a number so we use the digits 0 to 9 and then capital letters.

Base-12 uses only 12 digits (0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B) to form a number. A represents 10 and B is 11.
Place values from right to left are 1, 12^1, 12^2, 12^3, 12^4, …

Base-16 uses only 16 digits (0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F) to form a number.
A represents 10, B is 11, C is 12, D is 13, E is 14 and F represents 15.
Place values from right to left are 1, 16^1, 16^2, 16^3, 16^4, …