

## Place Value Numeration Systems

**Base-10** (Hindu-Arabic) uses 10 single digits (0, 1, 2, 3, 4, 5, 6, 7, 8, 9) to form a number.

It is a **place value system**, which means the position a digit is in has a place value based on the powers of 10. For example, 200 is 2 hundreds ( $10^2$ ) but 20 is 2 tens ( $10^1$ ) and 2 is 2 ones ( $10^0$ ).

Place value name	Million	Hundred-thousand	Ten-thousand	Thousand	Hundred	Ten	Ones
Power of ten	$10^6$	$10^5$	$10^4$	$10^3$	$10^2$	$10^1$	$10^0=1$

We can write a number in **expanded notation** by writing the face value times the place value. For example: 1,234,567 in expanded form is  $1*10^6 + 2*10^5 + 3*10^4 + 4*10^3 + 5*10^2 + 6*10^1 + 7*1$

**Counting:** When we count in base-10 we are adding 1 to the ones place value. Every time there is a “group” of 10 in any place value it is carried over to the next place value.

0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 (ten = 1 group of 10 and 0 ones)  
 11, 12, 13, 14, 15, 16, 17, 18, 19, 20 (19 = 1 group of 10 & 9 ones, 20 = 2 groups of 10 and 0 ones), ...,  
 91, 92, 93, 94 95 96 97 98, 99, 100 (93 = 9 groups of 10 & 3 ones, 100 = 1 group of 100 and 0 tens and 0 ones)

**For bases other than 10, a subscript at the end of the number indicates the base.**

**Base-5** uses only 5 digits (0, 1, 2, 3, 4) to form a number.

The **place value** of a digit is based on **powers of 5**. The place value powers of 5 increase as we “read” the numeral from *right to left*.

For example, the base-5 numeral 12304<sub>5</sub> in expanded form is  $1*5^4 + 2*5^3 + 3*5^2 + 0*5^1 + 4*1$

**Counting:** When we count in base-5 we are adding 1 to the digit in the ones place value. Every time there is a “group” of 5 in a place value it is carried over to the next larger place value.

0<sub>5</sub>, 1<sub>5</sub>, 2<sub>5</sub>, 3<sub>5</sub>, 4<sub>5</sub>, 10<sub>5</sub>, (10<sub>5</sub> = 1 group of 5 and 0 ones)  
 11<sub>5</sub>, 12<sub>5</sub>, 13<sub>5</sub>, 14<sub>5</sub>, 20<sub>5</sub>,  
 (11<sub>5</sub> = 1 group of 5 & 1 one, 12<sub>5</sub> = 1 group of 5 & 2 ones, 13<sub>5</sub> = 1 group of 5 & 3 ones,  
 14<sub>5</sub> = 1 group of 5 and 4 ones and 20<sub>5</sub> = 2 groups of 5 and zero ones), ...,  
 41<sub>5</sub>, 42<sub>5</sub>, 43<sub>5</sub>, 44<sub>5</sub>, 100<sub>5</sub>, (100<sub>5</sub> = 1 group of 5<sup>2</sup>, zero 5s and 0 ones), ...

**To convert a base-5 number to base-10:** multiply each face value by its place value.

For example, the base-5 numeral 12304<sub>5</sub> in base-10 is  
 $1*5^4 + 2*5^3 + 3*5^2 + 0*5^1 + 4*1 = 1*625 + 2*125 + 3*25 + 0*5 + 4*1 = 954$

**To convert a base-10 to base-5:** we need to divide by powers of 5. Start with the largest possible power of 5 that is less than the base-10 number.

For example, write 2146 in base-5

Powers of 5	$5^5=3125$	$5^4=625$	$5^3=125$	$5^2=25$	$5^1=5$	$5^0=1$
	bigger	$2146 \div 625$	$271 \div 125$	$21 \div 25$	$21 \div 5$	$1 \div 1$
<b>quotient</b>	than 2146	<b>3</b>	<b>2</b>	<b>0</b>	<b>4</b>	<b>1</b>
remainder		271	21	21	1	0

The equivalent base-5 number is the quotients: **32041<sub>5</sub>**

The above processes can be used in any number base.  
Remember that the place values need to represent the number base being used

**Base-3** uses only 3 digits (0, 1, 2) to form a number.

Place values from right to left are 1,  $3^1$ ,  $3^2$ ,  $3^3$ ,  $3^4$ , ...

**Base-4** uses only 4 digits (0, 1, 2, 3) to form a number.

Place values from right to left are 1,  $4^1$ ,  $4^2$ ,  $4^3$ ,  $4^4$ , ...

**Base-6** uses only 6 digits (0, 1, 2, 3, 4, 5) to form a number.

Place values from right to left are 1,  $6^1$ ,  $6^2$ ,  $6^3$ ,  $6^4$ , ...

**Base-8** uses only 8 digits (0, 1, 2, 3, 4, 5, 6, 7) to form a number.

Place values from right to left are 1,  $8^1$ ,  $8^2$ ,  $8^3$ ,  $8^4$ , ...

**Base-9** uses only 9 digits (0, 1, 2, 3, 4, 5, 6, 7, 8) to form a number.

Place values from right to left are 1,  $9^1$ ,  $9^2$ ,  $9^3$ ,  $9^4$ , ...

For **bases larger than 10** we need more than 10 *single* digits to form a number so we use the digits 0 to 9 and then capital letters.

**Base-12** uses only 12 digits (0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B) to form a number. A represents 10 and B is 11.

Place values from right to left are 1,  $12^1$ ,  $12^2$ ,  $12^3$ ,  $12^4$ , ...

**Base-16** uses only 16 digits (0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F) to form a number.

A represents 10, B is 11, C is 12, D is 13, E is 14 and F represents 15.

Place values from right to left are 1,  $16^1$ ,  $16^2$ ,  $16^3$ ,  $16^4$ , ...