Polynomial Functions, Rational Functions and Transformations

Polynomial Functions

We get **polynomial functions** by adding or subtracting power functions with positive integer powers.

Examples: $f(x) = x^2 + 3x - 6, \quad g(x) = x^3 - x^4 + 7x^2 - 2$

Since we can plug any numbers into polynomials, the domain is all real numbers $(-\infty, \infty)$

In polynomials, we look for **roots = x-intercepts = zeroes**

The highest power of a polynomial is called its **degree**.

For polynomials the **number of roots $\leq$ degree**

Another interesting feature of polynomials are the high and low points.

A high point is called a **maximum** (the plural is **maxima**), and low point is classed a **minimum** (the plural is **minima**).

An **extremum** (plural **extrema**) is either a max or a min

In polynomials, the **number of extrema $\leq$ degree – 1**

Rational Functions

We get **rational functions** by dividing polynomials.

Examples:

$$f(x) = \frac{2x - 8}{x + 3}, \quad g(x) = \frac{2x - 8}{x^2 - 16}, \quad h(x) = \frac{x^3}{x^2 + x - 20}$$

With rational functions we see interesting properties where the numerator = 0 or the denominator = 0

We can see x-intercepts and singularities. A **singularity** is where the functions goes up to $+\infty$ or down to $-\infty$

In the function *f(x)* above: The function f has an x-intercept where the numerator = 0 ($x = 4$). The function f has a singularity where the denominator = 0 ($x = -3$)

The singularity also shows a **vertical asymptote**. An **asymptote** is a line, where the graph gets infinitesimally close to the line.

Rational function may also have **horizontal asymptotes**. The function f above has a horizontal asymptote at $y = 2$

Some rational functions have "**holes**" in their graphs – missing points. This may occur where both the numerator = 0 and the denominator = 0. For example, the function g above has a hole at $x = 4$.

Some rational functions have **oblique asymptotes**, which are lines, but not vertical or horizontal. For example the function h above has an oblique asymptote

Rational Function Facts (num = numerator, denom = denominator)

- The zeroes occur where num = 0 but denom $\neq 0$
- Vertical asymptotes occur where num $\neq 0$ but denom = 0
- Holes occur where both num = 0 and denom = 0
- If degree num $\leq$ degree denom, you should see a horizontal asymptote
- If degree num $>$ degree denom, you should see an oblique asymptote
- For horizontal or oblique asymptotes, compute $\frac{\text{Highest Power in Num}}{\text{Highest Power in Denom}}$

Polynomial Functions, Rational Functions and Transformations

Transforming Graphs

There are methods of stretching, shrinking, and moving function graphs.

Examples: Graph the functions $y = x^2$, $y = 3x^2$, $y = \frac{1}{2}x^2$

Graph the functions $y = 3x^2$, $y = -3x^2$

Facts About Multiplying Functions by Constants

1. If $k > 1$, then we graph $k\,f(x)$ by stretching the graph of $f(x)$ vertically
2. If $0 < k < 1$, then we graph $k\,f(x)$ by shrinking the graph of $f(x)$ vertically
3. To get the graph of $-f(x)$, we flip the graph of $f(x)$ over the x-axis

Examples: Graph the functions $y = x^2$, $y = x^2 - 4$, $y = x^2 + 3$

Facts About Adding or Subtracting a Number to a Function: Assume b is a positive number.

1. We get the graph of $f(x) + b$ by moving the graph of $f(x)$ up by b-units
2. We get the graph of $f(x) - b$ by moving the graph of $f(x)$ down by b-units

Examples: Graph the functions $y = x^2$, $y = (x-4)^2$, $y = (x+3)^2$

Facts About Adding or Subtracting a Number to a Function: Assume a is a positive number.

1. We get the graph of $f(x - a)$ by moving the graph of $f(x)$ RIGHT by a-units
2. We get the graph of $f(x + a)$by moving the graph of $f(x)$ LEFT by a-units