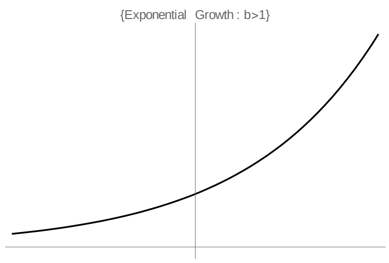
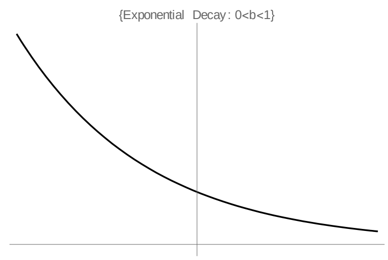
Exponential Functions, Inverse Functions, Logarithmic Functions,

Exponential Functions

Exponential functions have the independent variable in the exponent. The base is a constant.
General Formula: $y = b^x$ for $b > 0$
Examples: Graph $y = 2^x$, $y = \left(\frac{1}{3}\right)^x$
Depending on the base, these models are usually called **exponential growth** or **exponential decay**.
Exponential functions have a horizontal asymptote at $y = 0$ (the x-axis), and a y-intercept at (0, 1)
The general graphs, using the general formula, for these functions are





The Important Number e and the Important Function $f(x) = e^x$

Since the number e was first discovered by Napier in 1618, many different formulas have been used to describe it.
Here is one formula to get e: on your calculator, create a function

$$y = \left(1 + \frac{1}{x}\right)^x$$

Then look at the values (y-coordinates) when x gets really big – over 100,000. You can do this with a table.
For very large x, this function gets closer and closer to a height of $e = 2.718281828459 \ldots$ (an infinite decimal which doesn't repeat).
On your calculator, you can also get e by calculating e^1
A very important exponential function is $f(x) = e^x$. Graph it.
In some books and in some software, this is written $Exp[x]$.

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Inverse Functions

Example: The conversion function for degrees Fahrenheit (x) into degrees Celsius (y) is $y = \frac{5}{9}(x - 32)$

Make a table which includes $x = 32$, 50, 80, 100, 212

Sometimes, we also want to convert °C into °F. The original formula gives us $C = \frac{5}{9}(F - 32)$

We now need to solve for F. Do the algebra and you get $F = \frac{9}{5}C + 32$

Fact: A function $y = f(x)$ sometimes has an inverse function. We obtain it by swapping the x and y and then solving for y.

The **symbol for the inverse** of the function f is written $\boldsymbol{f^{-1}(x)}$. **THIS DOES NOT MEAN RECIPROCAL!!!!!**

Example: $f(x) = \frac{5}{9}x - \frac{160}{9}$. Find $f^{-1}(x)$.

$$y = \frac{5}{9}x - \frac{160}{9}$$

swap *x* and *y*: $x = \frac{5}{9}y - \frac{160}{9}$

solve for *y*: $x + \frac{160}{9} = \frac{5}{9}y$

$\frac{9}{5}\left(x + \frac{160}{9}\right) = y$ or $y = \frac{9}{5}x + 32$

In this example, try $x = 77$ in the original function and convert the answer "backwards".

What do you get from $(f^{-1} \circ f)(77)$? Also find $(f \circ f^{-1})(25)$ and

Fact: If f has an inverse function, then $(f \circ f^{-1})(x) = x$ and $(f^{-1} \circ f)(x) = x$

Inverse Functions Continued

A function is called **two-to-one ($\mathbf{2 - 1}$)** if at least two x's give the same y.

A function is called **one-to-one ($\mathbf{1 - 1}$)** if it is not $2 - 1$.

Fact: One-to-One functions have inverses

The Horizontal Line Test for Inverses of Functions (HLT): Assume y is a function of x. If a horizontal line intersects the graph of the function more than once, then the function does NOT have an inverse.

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Logarithmic Functions

Logarithmic functions are the inverses of exponential functions. We again work with positive bases ($b > 0$)

Example: Graph $f(x) = 2^x$ and make a table for the function with $x = -2, -1, 0, 1, 2$

We can see that the graph passes the HLT.

On the table swap the x and y. Graph the points on the new table

This is the same as writing $x = 2^y$. We write this as $y = \log_2(x)$ or $y = \log_2 x$

When doing logarithms, think about switching x and y from an exponential function and looking for an exponent

Facts: $f(x) = b^x \Leftrightarrow f^{-1}(x) = \log_b(x)$ which can be written $x = b^y \Leftrightarrow y = \log_b(x)$

Examples: Find $\log_2 32,\ \log_{10} 10,\ \log 10000,\ \ln(e^{-2}),\ \log(-5),\ \log(0)$

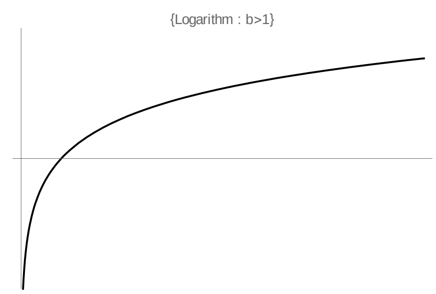
Shorthand: $\log x$ means $\log_{10} x$ $\ln x$ means $\log_e x$

Facts:
1. The answer of a logarithm is an exponent
2. We cannot take a log of a negative number
3. We cannot take a log of zero

Graphs of Functions and Inverses

Example: Graph the functions $y = 10^x,\ y = \log x,\ y = x$

We see that the graphs of the exponential and logarithm are symmetric = mirror images.



Fact: For $b > 1$, the graph of $f(x) = \log_b(x)$ has an x-intercept at $x = 1$ and a vertical asymptote at $x = 0$ (the y-axis). The general shape is shown on the right:

Fact About Graphs of $f(x)$ and $f^{-1}(x)$

The graphs of a function and its inverse are symmetric around the line $y = x$

Properties of Logarithms:

Because of the properties of exponents, we can get related properties of logs

Exponential Property	Logarithmic Property (Work for Any Base)
$x^p x^q = x^{p+q}$	$\log(AB) = \log A + \log B$
$\frac{x^p}{x^q} = x^{p-q}$	$\log\left(\frac{A}{B}\right) = \log A - \log B$
$(x^p)^q = x^{pq}$	$\log(A^q) = q(\log A)$
	$\log_b(x) = \frac{\log_c(x)}{\log_c(b)}$

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