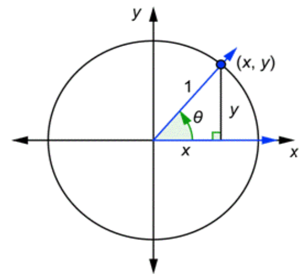
PreCalculus and Calculus I – Angles, Radians, Trigonometric Functions, Trig Identities

Angles

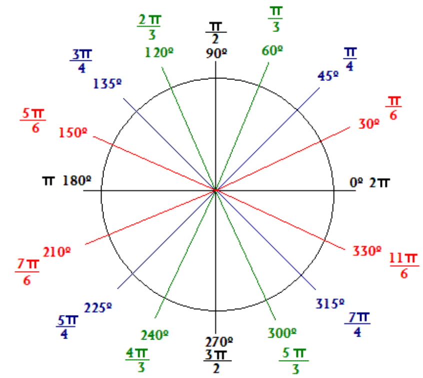
We create an angle by drawing two intersecting lines or two radii in a circle. The **standard position** of the angle:

- Place the center of the circle at the origin $(0,0)$
- Draw one radius on the positive x-axis
- Measure the angle from this first radius in a counter-clockwise direction



Degrees and Radians

- We form 1 **degree** $(1°)$ by using the angle which gives $\frac{1}{360}$ of the circle
- The **unit circle** has radius 1
- On the unit circle, the angle formed by arc length = 1 is called 1 **radian**
- In general, a radian is measured by arc length on the unit circle
- Because of the formula for circumference, the full circle gives 2π radians, so π radians $= 180°$
- We can create a circle with many special angles – shown on right



Trigonometric Functions

We place θ in the unit circle with endpoint (x, y).

We get 6 trig functions: Sine, Cosine, Tangent, Cotangent, Secant, Cosecant

$\cos(\theta) = x$	$\sin(\theta) = y$
$\sec(\theta) = \frac{1}{x} = \frac{1}{\cos\theta}$	$\csc(\theta) = \frac{1}{y} = \frac{1}{\sin\theta}$
$\tan(\theta) = \frac{y}{x} = \frac{\sin\theta}{\cos\theta}$	$\cot(\theta) = \frac{x}{y} = \frac{\cos\theta}{\sin\theta}$

By the Pythagorean Theorem, we get that $\cos^2\theta + \sin^2\theta = 1$

Divide by $\cos^2\theta$ to get $1 + \tan^2\theta = \sec^2\theta$

Divide by $\sin^2\theta$ to get $\cot^2\theta + 1 = \csc^2\theta$

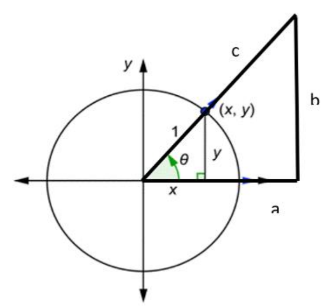
Trigonometric Functions and Right Triangles

Place any right triangle's angle in standard position and draw the unit circle as we did

Then, the given triangle $(a - b - c)$ is **similar** to the $x - y - 1$ triangle. This gives us:

- $\sin\theta = y = \frac{y}{1} = \frac{b}{c} = \frac{\text{opposite}}{\text{hypotenuse}}$
- $\cos\theta = x = \frac{x}{1} = \frac{a}{c} = \frac{\text{adjacent}}{\text{hypotenuse}}$
- $\tan\theta = \frac{y}{x} = \frac{b}{a} = \frac{\text{opposite}}{\text{adjacent}}$

This is usually written **SOH-CAH-TOA**



Graphs of Trig Functions

Ex: Graph $y = \sin x$

- To get the graph of $y = \cos x$ we shift the graph of the sine to the left by $\frac{\pi}{2}$. That is, $\cos x = \sin\left(x + \frac{\pi}{2}\right)$
- Sin, Cos, Sec, Csc have periods of 2π
- Tan and Cot have periods of π

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More on Trig Graphs

- We can shift and stretch trig functions just like we did for other functions. The general forms are
$$y = A\sin\big(B(x+C)\big) + D, \quad y = A\cos\big(B(x+C)\big) + D$$
  - $|A|$ gives the amplitude of the graph = distance from the middle of the graph to the max or min
  - $x + C$ tells us to shift the graph right if C is negative, and shift left if C is positive
  - $\frac{2\pi}{|B|}$ gives the period of the graph
  - D is the vertical shift of the graph

Trig Identities: Each of these identities can be proven correct by using triangles, circles, or graphs.

- $\cos^2\theta + \sin^2\theta = 1$
- $1 + \tan^2\theta = \sec^2\theta$
- $\cot^2\theta + 1 = \csc^2\theta$
- $\cos\theta = \sin\left(\theta + \frac{\pi}{2}\right)$
- $\cos(-\theta) = \cos\theta$
- $\sin(-\theta) = -\sin\theta$
- $\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$
- $\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$
- $\sin(2\theta) = 2\sin\theta\cos\theta$
- $\sin(2\theta) = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$

Inverse Trig Functions

If we graph $y = \sin x$, we see that it fails the Horizontal Line Test – it is not 1-1.
We can force $f(x) = \sin x$ to have an inverse function $f^{-1}(x)$ by "**restricting its domain**"
Ex: Graph $y = \sin x$ on the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
This new graph has an inverse – swap x and y to get $f^{-1}(x) = \sin^{-1} x$

Ex: Graph $y = \cos x$ on the interval $[0, \pi]$
This new graph has an inverse – swap x and y to get $f^{-1}(x) = \cos^{-1} x$

Ex: Graph $y = \tan x$ on the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
This new graph has an inverse – swap x and y to get $f^{-1}(x) = \tan^{-1} x$