Calculus I - Derivatives Rules

The **Average Rate of Change** of the function f on $[a, x]$ or the **slope of the secant line** on the graph of f over $[a, x]$:

$$m_{\text{sec}} = \frac{f(x) - f(a)}{x - a}$$

The **Derivative** of f at $x = a$ is the **Instantaneous Rate of Change** at $x = a$ or the **slope of the tangent line** to its graph at $x = a$:

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a} = \lim_{h \to 0} \frac{f(a + h) - f(a)}{h} = m_{\tan}$$

If the limit exists, we say that f is **differentiable** at a.

When we think of the derivative of $y = f(x)$ as a function, we write

$$f'(x) = \frac{dy}{dx} = \frac{d}{dx}[f(x)]$$

Higher Derivatives

$$f''(x) = \frac{d}{dx}[f'(x)] = \frac{d^2y}{dx^2}, \quad f'''(x) = \frac{d}{dx}[f''(x)] = \frac{d^3y}{dx^3}, \quad f^{(4)}(x) = \frac{d}{dx}[f'''(x)] = \frac{d^4y}{dx^4}, \quad \ldots \quad f^{(n)}(x) = \frac{d}{dx}\left[f^{(n-1)}(x)\right] = \frac{d^ny}{dx^n}$$

Derivative Rules

1. $\frac{d}{dx}[c] = 0$
2. $\frac{d}{dx}[x^n] = nx^{n-1}$
3. $\frac{d}{dx}[e^x] = e^x$
4. $\frac{d}{dx}[b^x] = b^x \ln b$
5. $\frac{d}{dx}[\ln|x|] = \frac{1}{x}$
6. $\frac{d}{dx}[\log_b|x|] = \frac{1}{x \ln b}$
7. $\frac{d}{dx}[\sin x] = \cos x$
8. $\frac{d}{dx}[\cos x] = -\sin x$
9. $\frac{d}{dx}[\tan x] = \sec^2 x$
10. $\frac{d}{dx}[\cot x] = -\csc^2 x$
11. $\frac{d}{dx}[\sec x] = \sec x \tan x$
12. $\frac{d}{dx}[\csc x] = -\csc x \cot x$
13. $\frac{d}{dx}[\sin^{-1} x] = \frac{1}{\sqrt{1 - x^2}}$
14. $\frac{d}{dx}[\cos^{-1} x] = -\frac{1}{\sqrt{1 - x^2}}$
15. $\frac{d}{dx}[\tan^{-1} x] = \frac{1}{1 + x^2}$
16. $\frac{d}{dx}[\cot^{-1} x] = -\frac{1}{1 + x^2}$
17. $\frac{d}{dx}[\sec^{-1} x] = \frac{1}{|x|\sqrt{x^2 - 1}}$
18. $\frac{d}{dx}[\csc^{-1} x] = -\frac{1}{|x|\sqrt{x^2 - 1}}$
19. $\frac{d}{dx}[c \cdot f(x)] = c \cdot f'(x)$
20. $\frac{d}{dx}[f(x) + g(x)] = f'(x) + g'(x)$

PRODUCT RULE

21. $\frac{d}{dx}[f(x)g(x)] = f(x)g'(x) + f'(x)g(x)$

QUOTIENT RULE

22. $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$

CHAIN RULE

23. $\frac{d}{dx}[f(g(x))] = f'(g(x))g'(x)$
24. $\frac{d}{dx}[f^{-1}(x)] = \frac{1}{f'\left(f^{-1}(x)\right)}$