

## Reply

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Doswell (2006) was confused by a step in Cohen and Schultz (2005), and so he produced a different derivation of our results without employing the complex notation we used in our paper. It is heartening to know that Doswell (2006) showed that the same answer could be derived in more than one way. We thank him for his efforts.

We appreciate the opportunity to clarify our derivation. In this reply, we include the missing steps between Equations (2) and (3) [here also numbered (2) and (3) for consistency], which were omitted from Cohen and Schultz (2005) for brevity.

We start with the second paragraph in section 2 of Cohen and Schultz (2005).

To simplify analysis and interpretation, only horizontal motions are considered. To represent the time dependence of the separation in two dimensions between these two fluid parcels, the separation vector for a small period of time  $\delta\mathbf{r}(t)$  is written in terms of a growth rate  $\sigma$  and rotation rate  $\omega$ :

$$\delta\mathbf{r}(t) = \delta\mathbf{r}(0) \exp[(\sigma + i\omega)t]. \quad (2)$$

Expressing  $\delta\mathbf{r}(0)$  as  $|\delta\mathbf{r}(0)| \exp(i\phi_0)$ , where  $\phi_0$  is the initial orientation of the separation vector, and breaking apart the exponential into two expressions yields

$$\delta\mathbf{r}(t) = |\delta\mathbf{r}(0)| \exp(i\phi_0) \exp(\sigma t) \exp(i\omega t).$$

Using Euler's equation,  $\exp(i\phi) = \cos\phi + i \sin\phi$ , twice results in

$$\begin{aligned} \delta\mathbf{r}(t) &= |\delta\mathbf{r}(0)| \exp(\sigma t)(\cos\phi_0 + i \sin\phi_0)(\cos\omega t + i \sin\omega t) \\ &= |\delta\mathbf{r}(0)| \exp(\sigma t)[(\cos\phi_0 \cos\omega t - \sin\phi_0 \sin\omega t) \\ &\quad + i(\sin\phi_0 \cos\omega t + \cos\phi_0 \sin\omega t)]. \end{aligned}$$

Employing trigonometric identities results in

$$\delta\mathbf{r}(t) = |\delta\mathbf{r}(0)| \exp(\sigma t)[\cos(\omega t + \phi_0) + i \sin(\omega t + \phi_0)].$$

Substituting  $|\delta\mathbf{r}| = |\delta\mathbf{r}(0)| \exp(\sigma t)$  and replacing  $(\omega t + \phi_0)$  with  $\phi$  yields

$$\delta\mathbf{r}(t) = |\delta\mathbf{r}|(\cos\phi + i \sin\phi).$$

Substituting  $\delta x = |\delta\mathbf{r}| \cos\phi$  and  $\delta y = |\delta\mathbf{r}| \sin\phi$  then yields

$$\delta\mathbf{r}(t) = \delta x + i\delta y. \quad (3)$$

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### REFERENCES

- Cohen, R. A., and D. M. Schultz, 2005: Contraction rate and its relationship to frontogenesis, the Lyapunov exponent, fluid trapping, and airstream boundaries. *Mon. Wea. Rev.*, **133**, 1353–1369.
- Doswell, C. A., III, 2006: Comments on “Contraction rate and its relationship to frontogenesis, the Lyapunov exponent, fluid trapping, and airstream boundaries.” *Mon. Wea. Rev.*, **134**, 2642–2643.

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