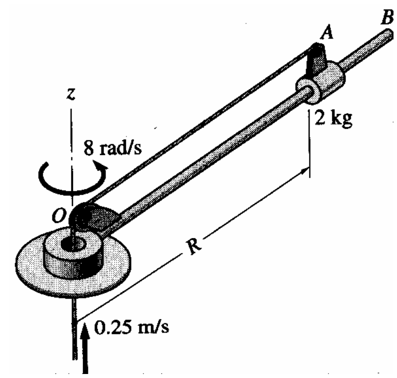
Phys202 - Dynamics - Spring 2007 HW9 Solutions

Problems 14.90, 14.116, 14.125, and 14.126

1 Problem 14.90



A motor rotates the rod OB about the z-axis at the constant angular speed of $\omega = 8\,\text{rad/s}$. The string attached to the 2 kg slider A is let out at the constant rate of 0.25 m/s. Determine the magnitude of the contact force between the slider and the rod when $R = 0.8\,\text{m}$. Neglect friction and the mass of rod OB.

Solution: All of the kinematics comes from the description, so we find the acceleration and use $\vec{F} = m\vec{a}$ to find the force. We know:

$$R = 0.8\,\text{m} \qquad \dot{R} = +0.25\,\text{m/s} \qquad \ddot{R} = 0$$

$$\theta \text{ doesn't matter} \qquad \omega = \dot{\theta} = 8\,\text{rad/s} \qquad \ddot{\theta} = 0$$

$$a_R = \ddot{R} - R\dot{\theta}^2 = -(0.8)(8)^2 = -51.2\,\text{m/s}^2$$

$$a_\theta = R\ddot{\theta} + 2\dot{R}\dot{\theta} = 2(0.25)(8) = 4\,\text{m/s}^2$$

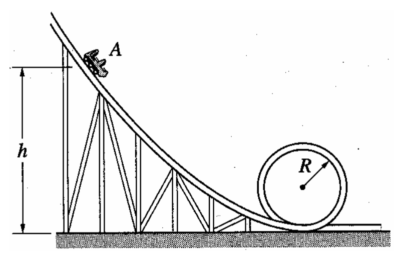
The force diagram of the slider would show the tension T pointing in the $-\hat{e}_R$ direction, gravity mg pointing in the $-\hat{k}$ direction, and the force of the rod $\vec{F}$ pointing in the θ and z directions. The $\theta$and z components are

$$\sum F_\theta = F_\theta = ma_\theta = (2\,\text{kg})\left(4\,\text{m/s}^2\right) = 8\,\text{N}$$

$$\sum F_z = F_z - mg = 0 \quad \rightarrow \quad F_z = mg = (2\,\text{kg})\left(9.81\,\text{m/s}^2\right) = 19.62\,\text{N}$$

So, the magnitude of the force is $F = \sqrt{F_\theta^2 + F_z^2} = \boxed{21.19\,\text{N}}$.

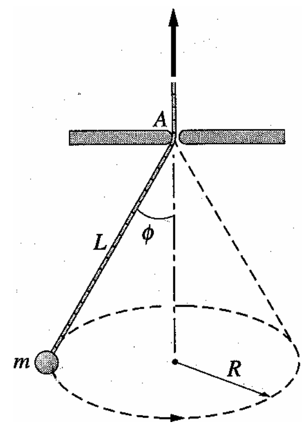
2 Problem 14.116



The car of a roller coaster travels with negligible friction on the track shown. If the car starts from rest at A, determine the smallest ratio h/R that is necessary for the car to stay in contact with the track.

Solution: This is a conservation of energy problem. Since the car starts from rest, the initial energy is $E_i = V_i = mgh$. To stick to the track, the normal force must be greater than zero. The acceleration at the top of the loop is $a = v^2/R$ downward. So, Newton's Second Law is $\sum F_z = -mg - N = -mv^2/R$ which means $v^2 = gR + NR/m$. The minimum speed occurs when $N = 0$, so $v^2 = gR$ is the threshold. The final energy is thus $E_f = \frac{1}{2}mv^2 + mgh_f = \frac{1}{2}m(gR) + mg(2R) = 2.5mgR$. The minimum height is thus $\boxed{h = 2.5R}$.

3 Problem 14.125



The mass m is attached to a string and swings in a horizontal circle (a **circular pendulum**) of radius $R = 0.25\,\text{m}$ when $L = 0.5\,\text{m}$. The length L is then shortened by pulling the string slowly through the hole A in the ceiling until the speed of the mass has doubled. Determine the corresponding values of (a) the radius R; and (b) the angle ϕ.

Solution: The first part of this problem is fairly easy. Since the moment (torque) of the tension is zero, angular momentum is conserved.

$$Rmv_i = h_i = h_f = R_f m\,(2v_i) \quad \rightarrow \quad R_f = R/2 = \boxed{0.125\,\text{m}} \text{ is the new radius.}$$

For the second part of the problem, we can't use conservation of energy, because the tension is doing work on the mass and both the kinetic and potential energy are changing. We can't use impulse-momentum, because the integral of the net force over time is zero (you should understand why). We can use $\vec{F} = m\vec{a}$. The tension T points in the $-R$ and z directions.

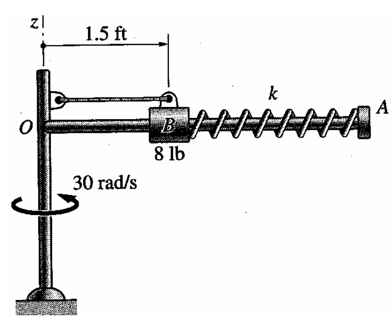
$$\sum F_z = T_z - mg = 0, \text{ so } T_z = T\cos\phi_f = mg$$

$$\sum F_R = T_R = ma_R = m\frac{v^2}{R}, \text{ so } T_R = -T\sin\phi_f = -\frac{mv^2}{R}$$

Dividing these, we get: $\tan\phi = \frac{v^2}{gR}$

This is true at any given instant. For the initial conditions, we get $v^2 = gR\tan\phi = \frac{gR^2}{\sqrt{L^2-R^2}} = 1.416$. For the final conditions, $\tan\phi = \frac{v_f^2}{gR_f} = \frac{4\cdot 1.416}{g(0.125)} = 4.619$, so $\boxed{\phi = 77.78°}$.

4 Problem 14.126



The rod OA is rotating freely about the z-axis with an angular speed of 30 rad/s. The spring is undeformed when the cord restraining the 8 lb collar B breaks. If the maximum displacement of the collar relative to the rod is 9 in, determine the stiffness k of the spring. Neglect friction andthe mass of rod OA.

Solution: Conservation of angular momentum tells us the final angular speed. $h_i = R_i mv_i = mR_i^2\dot{\theta}_i = h_f = R_f mv_f = mR_f^2\dot{\theta}_f$, with $R_i = 1.5\,\text{ft}$, $R_f = 2.25\,\text{ft}$, and $\dot{\theta}_i = 30\,\text{rad/s}$.

Since we know the initial and final (radial) velocities are zero, and we want the position, conservation of energy is the way to go. Since the spring is initially undeformed, it doesn't appear in E_i but it does appear in E_f, because $\delta_f = 0.75\,\text{ft}$.

$$E_i = \tfrac{1}{2}mv_i^2 = \tfrac{1}{2}m\left(R_i\dot{\theta}_i\right)^2 = E_f = \tfrac{1}{2}m\left(R_f\dot{\theta}_f\right)^2 + \tfrac{1}{2}k\delta_f^2$$

At this point, $\dot{\theta}_i$, $\dot{\theta}_f$, R_i, R_f, m, and δ_f are all known, so k can be solved for.