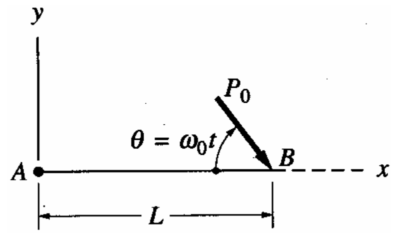
Phys202 - Dynamics - Spring 2007 HW8 Solutions

1 Problem 14.82

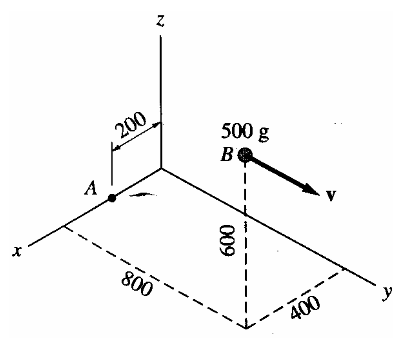


The force shown has the constant magnitude P_0 and the fixed point of application B, but its line of action rotates with the constant angular speed ω_0. Determine the angular impulse of the force about A for the time during which the force rotates from $\theta = 0$ to $\theta = 90°$.

Solution: The angular impulse is calculated from the definition. To make the integral easier, do a "u substitution". Since $\theta = \omega_0 t$, $d\theta = \omega_0 dt$. Remember that we always do calculus with trig functions in radians.

$$\vec{A}_{1\to 2} = \int_{t_1}^{t_2} \vec{r}\otimes\vec{F}\,dt = \int_0^{\pi/2} \left(L\hat{\imath}\right)\otimes\left(P_0\cos\theta\hat{\imath} - P_0\sin\theta\hat{\jmath}\right)\frac{d\theta}{\omega_0} = \frac{-P_0 L\hat{k}}{\omega_0}\int_0^{\pi/2}\sin\theta d\theta = \frac{-P_0 L}{\omega_0}\hat{k}$$

2 Problem 14.83



(Dimensions in mm)

The velocity of the 500 g particle at B is $\vec{v} = 6\hat{\imath} + 4\hat{\jmath} + 2\hat{k}$ m/s. Calculate the angular momentum of the particle about point A at this instant.

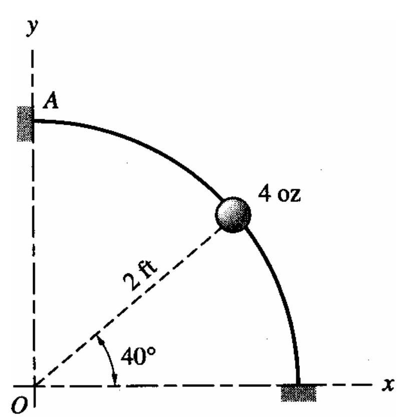
Solution: The angular momentum is $\vec{h}_A = \vec{r}\otimes m\vec{v}$. The displacement is from the center of rotation to the position of the particle, so

$$\vec{r} = \vec{r}_B - \vec{r}_A = 0.4\hat{\imath} + 0.8\hat{\jmath} + 0.6\hat{k} - 0.2\hat{\imath} = 0.2\hat{\imath} + 0.8\hat{\jmath} + 0.6\hat{k}$$

The angular momentum is

$$\begin{aligned}\hat{h}_A &= \left(0.2\hat{\imath} + 0.8\hat{\jmath} + 0.6\hat{k}\right)\otimes(0.5)\left(6\hat{\imath} + 4\hat{\jmath} + 2\hat{k}\right)\\ &= (0.5)\left((1.6-2.4)\,\hat{\imath} + (3.6-0.4)\,\hat{\jmath} + (0.8-4.8)\,\hat{k}\right)\\ &= -0.4\hat{\imath} + 1.6\hat{\jmath} - 2.0\hat{k}\,\mathrm{N\cdot m/s}\end{aligned}$$

3 Problem 14.84



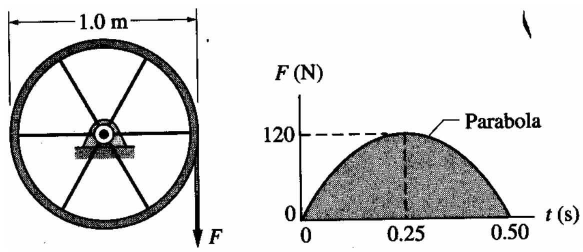
The 4 oz. bead is sliding down the circular wire. When the bead is in the position shown, its speed is 20 ft/s. For this instant, determine the angular momentum of the bead about (a) point O; and (b) point A.

Solution: The velocity vector is $\vec{v} = v\sin 40°\hat{\imath} - v\cos 40°\hat{\jmath} = 15.32\hat{\imath} - 12.86\hat{\jmath}$ m/s. For $\vec{h}_O$, $\vec{r} = (2\,\mathrm{ft})\cos 40°\hat{\imath} + (2\,\mathrm{ft})\sin 40°\hat{\jmath}$. We can use the cross product in vector form, but since the displacement and velocity are perpendicular, the magnitude is simply $h_O = rmv = (2\,\mathrm{ft})\left(0.25\,\mathrm{lb}/32.2\,\mathrm{ft/s^2}\right)(20\,\mathrm{ft/s}) = 0.311\,\mathrm{lb\cdot ft/s}$. The direction is $-\hat{k}$ by the right-hand rule.

For (b) the angular momentum about point A, $\vec{r} = (2\,\mathrm{ft})\cos 40°\hat{\imath} + (2\,\mathrm{ft})(\sin 40° - 1)\,\hat{\jmath}$. The angular momentum is

$$\begin{aligned}\vec{h}_A &= (2\,\mathrm{ft})\left(\cos 40°\hat{\imath} + (\sin 40° - 1)\,\hat{\jmath}\right)\otimes\left(0.25\,\mathrm{lb}/32.2\,\mathrm{ft/s^2}\right)(20\,\mathrm{ft/s})\left(\sin 40°\hat{\imath} - \cos 40°\hat{\jmath}\right)\\ &= (0.311\,\mathrm{lb\cdot ft}/s)\left(-\cos^2 40°\hat{k} + \sin 40°\left(-\hat{k}\right) - \sin 40°\left(-\hat{k}\right)\right) = -0.311\,(1-\sin 40°) = -0.111\hat{k}\,\mathrm{lb\cdot ft/s}\end{aligned}$$

4 Problem 14.87



A rope wound around the rim of the wheel is pulled by force F, which varies with time as shown. Calculate the angular impulse of F about the center of the wheel during the time interval $t = 0$ to $t = 0.5\,\mathrm{s}$.

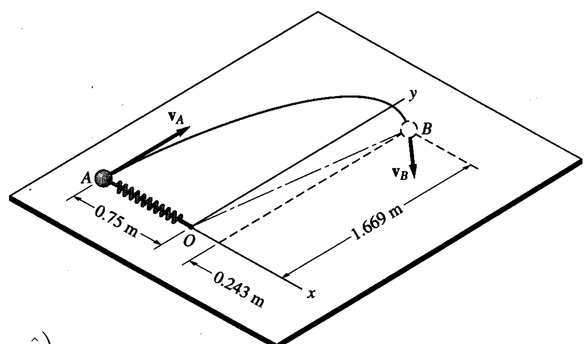
Solution: The force can be parameterized by $F(t) = a\,(t - t_0)^2 + F_0$. The time shift is $t_0 = 0.25\,\mathrm{s}$, while the vertical shift is $F_0 = 120\,\mathrm{N}$. By setting $F(0) = 0 = a\,(0.25)^2 + 120$, we find that $a = -120/0.25^2 = -1920\,\mathrm{N/s^2}$.

$$F(t) = -1920\,(t - 0.25)^2 + 120\,\mathrm{N}$$

The angular impulse is

$$A_{1\to 2} = \int_0^{0.5} (0.5\,\mathrm{m})\left(120 - 1920\,(t - 0.25)^2\right)dt = 30 - 320\left[(t - 0.25)^3\right]_0^{0.5} = 30 - 10 = \boxed{20\,\mathrm{N \cdot m/s}}$$

5 Problem 14.89



The particle, connected by a spring to the fixed point O, slides on the frictionless, horizontal table. The particle is launched at A with velocity v_A in the y-direction. If the velocity of the particle at B is $\vec{v}_B = 3.66\hat{\imath} - 5.72\hat{\jmath}\,\mathrm{m/s}$, determine v_A.

Solution: Since the spring exerts a central force (always pointed toward O), angular momentum is conserved.

$$(0.75)\,mv_A = h_i = h_f = \left(0.243\hat{\imath} + 1.669\hat{\jmath}\right)\otimes m\left(3.66\hat{\imath} - 5.72\hat{\jmath}\right)$$

$$v_A = (-1.390 - 6.109)\,/0.75 = 10.0\,\mathrm{m/s}$$