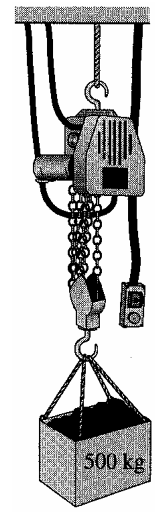
Phys202 - Dynamics - Spring 2007 HW7 Solutions

1 Problem 14.53



An electric hoist lifts a 500 kg mass at a constant speed of 0.76 m/s while consuming 4.5 kW of power. (a) Compute the efficiency of the hoist. (b) At what constant speed can the hoist lift a 750 kg mass with the same power consumption as in part (a)?

Solution: The power applied to the mass, which is the output power of the hoist, is

$$P_{\text{out}} = \vec{F} \cdot \vec{v} = mgv = (500\,\text{kg})\left(9.81\,\text{m/s}^2\right)(0.76\,\text{m/s}) = 3.728\,\text{kW}$$

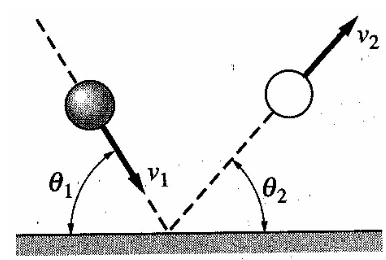
So, the efficiency is

$$\eta = \frac{P_{\text{out}}}{P_{\text{in}}} = \frac{3.728\,\text{kW}}{4.5\,\text{kW}} \times 100\% = \boxed{82.8\%}$$

For part (b), we can get the same P_{out}, so

$$v_2 = \frac{P_{\text{out}}}{m_2 g} = \frac{3728\,\text{W}}{(750\,\text{kg})\left(9.81\,\text{m/s}^2\right)} = \boxed{0.507\,\text{m/s}}$$

2 Problem 14.68



A 2.5 oz ball hits a frictionless, rigid, horizontal surface with a speed $v_1 = 30\,\text{ft/s}$ at the angle $\theta_1 = 70°$. The angle of rebound is $\theta_2 = 62°$. Compute (a) the speed of the ball immediately after the rebound; and (b) the resultant impulse acting on the ball during its time of contact with the surface.

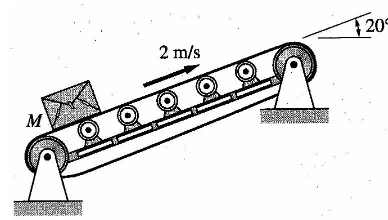
Solution: Since the wall is frictionless, the x component of the momentum p_x cannot change, so v_x is also constant.

$$\begin{aligned} v_x &= v_1 \cos\theta_1 = v_2 \cos\theta_2 \\ v_2 &= (30\,\text{ft/s})\,\frac{\cos 70°}{\cos 62°} = \boxed{21.86\,\text{ft/s}} \end{aligned}$$

To find the impulse, we need the y components before and after the collision with the wall. The mass is $m = \frac{(2.5\,\text{oz})}{(32.2\,\text{ft/s}^2)}\frac{(1\,\text{lb})}{(16\,\text{oz})} = 4.852 \times 10^{-3}\,\text{slug}$.

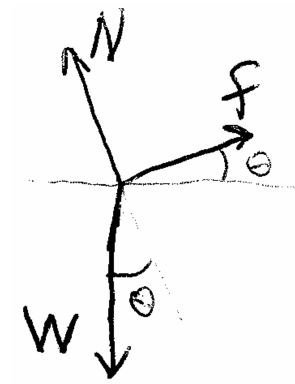
$$\vec{L}_{1\to 2} = \Delta\vec{p} = mv_2 \sin\theta_2 + mv_1 \sin\theta_1 = (0.004852\,\text{slug})\left((21.86\,\text{ft/s})\sin 62° + (30\,\text{ft/s})\sin 70°\right) = \boxed{0.230\,\text{lb}\cdot\text{s}}$$

3 Problem 14.120



A package of mass M is placed with zero velocity on a conveyor belt that is moving at 2 m/s. The coefficient of kinetic friction between the package and the belt is 0.4. Determine the distance that the package travels before it reaches the speed of the belt.

Solution: First, draw a force diagram to find the force of friction on the package. In the normal direction,



$$\sum F_N = N - W\cos\theta \quad \rightarrow \quad N = W\cos\theta = mg\cos\theta$$

In the direction of the belt,

$$\sum F_B = f - W\sin\theta = \mu_K N - mg\sin\theta = mg\left(\mu\cos\theta - \sin\theta\right)$$

It is this total force in the direction of the belt that is exerted over a distance. To find that distance, the work equals the change in kinetic energy.

$$\begin{aligned} F\Delta s &= U_{1\to 2} = \Delta T = \frac{1}{2}mv^2 \\ \Delta s &= \frac{mv^2}{2mg\left(\mu\cos\theta - \sin\theta\right)} = \frac{\left(2\,\mathrm{m/s}\right)^2}{2\left(9.81\,\mathrm{m/s^2}\right)\left(0.4\cos 20^\circ - \sin 20^\circ\right)} = \boxed{6.02\,\mathrm{m}} \end{aligned}$$

4 Problem 14.121

Determine the time required for the package in Problem 14.120 to reach the speed of the conveyor belt.

Solution: The force diagram and forces are the same as in 14.120. But this time use impulse-momentum.

$$\begin{aligned} mg\left(\mu\cos\theta - \sin\theta\right)\hat{\jmath}\,\Delta t &= \vec{F}\Delta t = \vec{L}_{1\to 2} = \Delta\vec{p} = mv\hat{\jmath} \\ \Delta t &= \frac{mv}{mg\left(\mu\cos\theta - \sin\theta\right)} = \frac{\left(2\,\mathrm{m/s}\right)}{\left(9.81\,\mathrm{m/s^2}\right)\left(0.4\cos 20^\circ - \sin 20^\circ\right)} = \boxed{6.02\,\mathrm{s}} \end{aligned}$$

Notice that during this period, the acceleration is constant. Therefore, the average speed is the average of the initial and final speeds.