Phys202 - Dynamics - Spring 2007 HW6 Solutions

1 Problem 14.2

Compute the work of a force $\vec{F} = (F_0/b^3)\left(xy^2\hat{i} + x^2y\hat{j}\right)$ as its point of application moves from the origin $\left(\vec{r}_1 = 0\hat{i} + 0\hat{j}\right)$ to the point $\vec{r}_2 = b\hat{i} + b\hat{j}$ along (a) the line $y = x$; and (b) the parabola $y = x^2/b$.

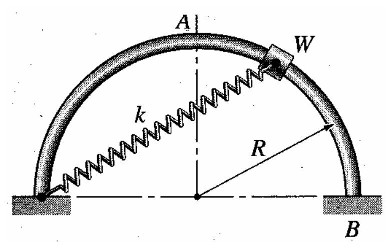
Solution: The work is just the line integral of the force. For part (a), $y = x$ and $dy = dx$:

$$W_{1\to 2} = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F}\cdot d\vec{r} = (F_0/b^3)\int_0^b \left(x^3\hat{i} + x^3\hat{j}\right)\cdot\left(dx\hat{i} + dx\hat{j}\right) = 2\,(F_0/b^3)\left[\frac{x^4}{4}\right]_0^b = \frac{2F_0b^4}{4b^3} = \boxed{\frac{F_0b}{2}}$$

For part (b), it's a little more complicated. $y = x^2/b$ and $d\vec{r} = dx\hat{i} + dy\hat{j}$, so we need $dy = 2xdx/b$.

$$\begin{aligned} W_{1\to 2} &= \int_{\vec{r}_1}^{\vec{r}_2} \vec{F}\cdot d\vec{r} = (F_0/b^3)\int_0^b \left(x\left(x^2/b\right)^2\hat{i} + x^2\left(x^2/b\right)\hat{j}\right)\cdot\left(dx\hat{i} + 2x/b\,dx\hat{j}\right) \\ &= (F_0/b^3)\int_0^b \left(\frac{x^5}{b^2} + \frac{x^4}{b}\frac{2x}{b}\right)dx = (F_0/b^3)\int_0^b \frac{3x^5}{b^2}\,dx \\ &= \frac{3}{b^2}\,(F_0/b^3)\left[\frac{x^6}{6}\right]_0^b = \frac{3F_0b^6}{6b^5} = \boxed{\frac{F_0b}{2}} \end{aligned}$$

2 Problem 14.4



The collar of weight W slides on a frictionless circular arc of radius R. The ideal spring attached to the collar has a free length $L_0 = R$ and stiffness k. When the slider moves from A to B, compute (a) the work done by the spring; and (b) the work done by the weight.

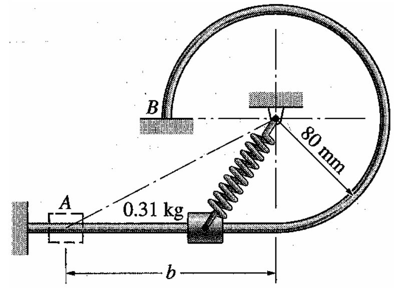
Solution: I'm using polar coordinates. The length of the spring at any given point is a triangle with sides $L_x = R(1+\cos\theta)$ and $L_y = R\sin\theta$. The work done by the spring is just the negative of the change in spring potential energy, $V_{\text{spring}} = \frac{1}{2}k\delta^2 = \frac{1}{2}k(L - L_0)^2$.

$$U_{\text{spring},1\to 2} = \frac{1}{2}k\delta_1^2 - \frac{1}{2}k\delta_2^2 = \frac{1}{2}k\left(\sqrt{2}R - R\right)^2 - \frac{1}{2}k(2R - R)^2 = \frac{1}{2}kR^2\left(\left(\sqrt{2}-1\right)^2 - 1\right) = \boxed{-0.414kR^2}$$

The work done by gravity is minus the change in the gravitational potential energy, $V_{\text{gravity}} = mgh = Wh$.

$$U_{\text{gravity},1\to 2} = Wh_1 - Wh_2 = \boxed{WR}$$

3 Problem 14.12

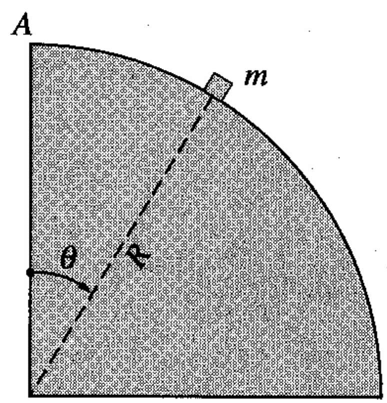


The 0.31 kg mass slides on a frictionless wire that lies in the vertical plane. The ideal spring attached to the mass has a free length of 80 mm and its stiffness is 120 N/m. Calculate the smallest value of the distance b if the mass is to reach the end of the wire at B after being released from A.

Solution: If the wire is to reach B, it must pass the point at the top of the loop. Use this for a conservation of energy analysis. At both points, the kinetic energy can be zero.

$$\begin{aligned}
E_i &= E_f \\
\frac{1}{2}k\left(L_i - L_0\right)^2 + mgh_i &= \frac{1}{2}k\left(L_f - L_0\right)^2 + mgh_f \\
\frac{1}{2}k\left(\sqrt{b^2+R^2} - L_0\right)^2 &= \frac{1}{2}k\left(R - L_0\right)^2 + 2mgR \\
\left(\sqrt{b^2+R^2} - L_0\right)^2 &= \overbrace{\left(R - L_0\right)^2}^{=0} + 4mgR/k \\
\sqrt{b^2+R^2} &= \sqrt{4mgR/k} + L_0 \\
b &= \sqrt{\left(\sqrt{4mgR/k} + L_0\right)^2 - R^2} \\
&= \boxed{0.150\,\mathrm{m}}
\end{aligned}$$

4 Problem 14.36



The particle of mass m is at rest at A when it is slightly displaced and allowed to slide down the cylindrical surface of radius R. Neglecting friction, determine (a) the speed of the particle as a function of the angle θ; and (b) the value of θ when the particle leaves the surface.

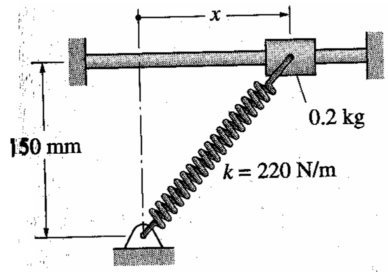
Solution: This is easy by conservation of energy.

$$\begin{aligned}
E_i &= E_f \\
mgh_i + \frac{1}{2}mv_i^2 &= \frac{1}{2}mv_f^2 + mgh_f \\
mgR &= \frac{1}{2}mv^2 + mgR\cos\theta \\
v &= \boxed{\sqrt{2gR\left(1 - \cos\theta\right)}}
\end{aligned}$$

Finding the point of separation comes from using the sum of forces in the radial direction and setting the normal force to $N = 0$. The acceleration is $a_R = -v^2/R$.

$$\begin{aligned}
N - mg\cos\theta = \sum F_R &= ma_r = -mv^2/R \\
g\cos\theta &= 2g\left(1 - \cos\theta\right) \\
3\cos\theta &= 2 \\
\theta &= \cos^{-1}\frac{2}{3} = \boxed{48.2^\circ}
\end{aligned}$$

5 Problem 14.39



The spring attached to the 0.2 kg collar has a stiffness of 220 N/m and a free length of 150 mm. If the collar is released from rest when $x = 150\,\text{mm}$, determine its acceleration as a function of x. Neglect friction.

Solution: This can be done with a simple force diagram and $F = ma$. The x component of spring force is $-x/L$ times the spring force.

$$k(L - L_0)\frac{-x}{L} = \sum F_x = ma_x = ma$$

$$k\left(\sqrt{x^2+y^2} - L_0\right)\frac{-x}{\sqrt{x^2+y^2}} = ma$$

$$a = -\frac{kx}{m}\frac{\sqrt{x^2+y^2} - L_0}{\sqrt{x^2+y^2}}$$

$$= -\frac{220x}{0.2}\cdot\frac{\sqrt{x^2+0.15^2} - 0.15}{\sqrt{x^2+0.15^2}} = \boxed{-1100x\left(1 - \frac{0.15}{\sqrt{x^2+0.0225}}\right)}$$