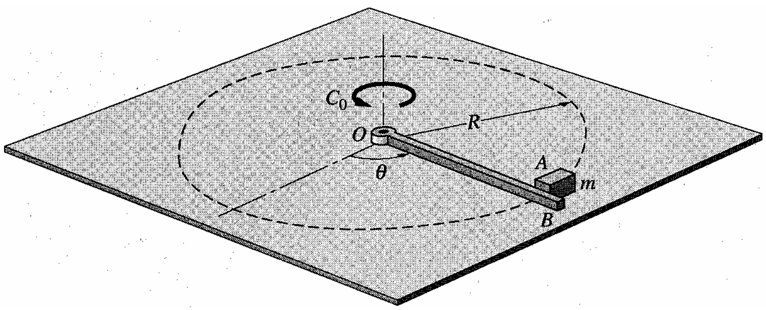
Phys202 - Dynamics - Spring 2007 HW5 Solutions

In this assignment, each student was given one problem to complete.

1 Problem 13.66 (Danny)

A block A of mass m and the rotating arm OB lie on a frictionless, horizontal table. The static coefficient of friction between the mass and the arm is μ. The system starts from rest at $\theta = 0$ under the action of the external couple C_0, which causes the angular acceleration $\ddot{\theta}$ of the arm to be constant. Determine the angle θ at the instant when A begins to slide relative to OB.



Solution: Use polar coordinates with circular motion ($\dot{R} = \ddot{R} = 0$). Then $a_R = -v^2/R = -R\dot{\theta}^2$ and $a_\theta = R\ddot{\theta}$. The radius is R. Since $\ddot{\theta}$is given as a constant, the kinematics are solved by integration using the initial conditions ($\dot{\theta}(0) = 0$ and $\theta(0) = 0$):

$$\dot{\theta}(t) = \int \ddot{\theta}\, dt = \ddot{\theta}t + C_1 = \ddot{\theta}t$$

$$\theta(t) = \int \dot{\theta}\, dt = \frac{1}{2}\ddot{\theta}t^2 + C_2 = \frac{1}{2}\ddot{\theta}t^2$$

Now that the kinnematics are known, we can find the forces. There is the circumferential force, which is the "Normal" force of contact on the block.

$$\sum F_\theta = F_N = ma_\theta = mR\ddot{\theta}$$

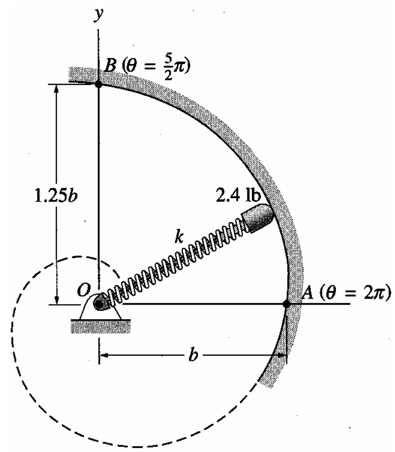
And the radial force on the block, which is the friction force.

$$\sum F_R = -F_F = -\mu F_N = -\mu mR\ddot{\theta} = ma_\theta = -mR\dot{\theta}^2 = -mR\left(\ddot{\theta}t\right)^2 = -mR\ddot{\theta}^2t^2$$

Solve for t^2 and combine with the equation for θ, we get

$$\begin{aligned} t^2 &= \frac{\mu mR\ddot{\theta}}{mR\ddot{\theta}^2} = \frac{\mu}{\ddot{\theta}} \\ &= \frac{2\theta}{\ddot{\theta}} \\ \theta &= \boxed{\mu/2} \end{aligned}$$

2 Problem 13.73 (James)



The 2.4 lb follower is attached to the end of a light telescopic rod that is pivoted at O. The follower is pressed against a frictionless spiral surface by a spring of stiffness $k = 8\,\mathrm{lb/ft}$ and free length $L_0 = 3\,\mathrm{ft}$. The equation of the spiral, which lies in the horizontal plane, is $R = b\theta/(2\pi)$, where $b = 1.2\,\mathrm{ft}$ and θ is in radians. Immediately after the rod is released from rest in position OA, determine (a) the angular acceleration $\ddot{\theta}$ of the rod; and (b) the contact force between the follower and the spiral surface.

Solution: Again, polar coordinates are easiest. We already know the relationship between R and θ:

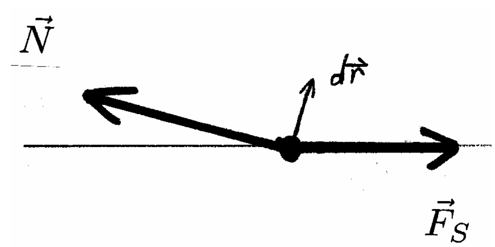
$$R = \frac{b}{2\pi}\theta \qquad \dot{R} = \frac{b}{2\pi}\dot{\theta} \qquad \ddot{R} = \frac{b}{2\pi}\ddot{\theta}$$

But we need to know how θ varies. This comes from Newton's Second Law. First the acceleration:

$$a_R = \ddot{R} - R\dot{\theta}^2 = \frac{b}{2\pi}\left(\ddot{\theta} - \theta\dot{\theta}^2\right) \qquad a_\theta = R\ddot{\theta} + 2\dot{R}\dot{\theta} = \frac{b}{2\pi}\left(\theta\ddot{\theta} + 2\dot{\theta}^2\right)$$

Now simplify with the initial conditions ($\theta = 2\pi$, $\dot{\theta} = 0$):

$$a_R = \frac{b}{2\pi}\left(\ddot{\theta}\right) \qquad a_\theta = \frac{b}{2\pi}\left(2\pi\ddot{\theta}\right)$$



The force diagram looks like the figure to the right. The radial direction is to the right, and the θ direction is up. The thin vector, $d\vec{r}$, is not a force. It marks the direction that the follower will move. We split $\vec{N}$ into radial and θ components and write Newton's Second Law. First the θ component:

$$N_\theta = \sum F_\theta = ma_\theta = mb\ddot{\theta}$$

And the radial component:

$$F_S + N_R = \sum F_R = ma_R = m\ddot{R} = \frac{mb}{2\pi}\ddot{\theta}$$

We have three unknowns (N_θ, N_R, and $\ddot{\theta}$) but only two equations so we need the constraints. This will provide a direction for $\vec{N}$, linking N_R and N_θ.

The direction along the surface is

$$d\vec{r} = d\left(R\hat{e}_R\right) = dR\,\hat{e}_R + R\,d\hat{e}_R = dR\,\hat{e}_R + R\hat{e}_\theta d\theta$$

We can divide this differential element by any small step to get a finite vector. Examples are $d\vec{r}/dt = \vec{v} = \dot{R}\,\hat{e}_R + R\dot{\theta}\,\hat{e}_\theta$ and $d\vec{r}/d\theta = \frac{dR}{d\theta}\,\hat{e}_R + R\,\hat{e}_\theta$. Declaring that $\vec{N}$ is perpendicular to $d\vec{r}/d\theta$, we get our third equation.

$$0 = \vec{N}\cdot\frac{d\vec{r}}{d\theta} = \left(N_R\hat{e}_R + N_\theta\hat{e}_\theta\right)\cdot\left(\frac{dR}{d\theta}\hat{e}_R + R\hat{e}_\theta\right) = \frac{b}{2\pi}N_R + b\,N_\theta = 0 \quad \rightarrow \quad N_R = -2\pi N_\theta$$

Now we can eliminate N_R and solve for N_θ:

$$F_S + N_R = F_S - 2\pi N_\theta = \frac{mb}{2\pi}\ddot{\theta} \quad \rightarrow \quad N_\theta = \frac{F_S}{2\pi} - \frac{mb}{4\pi^2}\ddot{\theta}$$

And plug this into the θ component of Newton's Second Law, noting that $F_S = k\delta = (8\,\mathrm{lb/ft})\,(3\,\mathrm{ft} - 1.2\,\mathrm{ft}) = 14.4\,\mathrm{lb}$ and $m = W/g = (2.4\,\mathrm{lb})\,/\left(32.2\,\mathrm{ft/s^2}\right) = 0.0745\,\mathrm{slug}$

$$\frac{F_S}{2\pi} - \frac{mb}{4\pi^2}\ddot{\theta} = mb\ddot{\theta} \quad \rightarrow \quad \ddot{\theta} = \frac{F_S/\left(2\pi\right)}{mb\left(1 + 1/\left(4\pi^2\right)\right)} = \frac{14.4/\left(2\pi\right)}{\left(0.0745\,\mathrm{slug}\right)\left(1.2\,\mathrm{ft}\right)\left(1.025\right)} = 25\,\mathrm{s}^{-2}$$

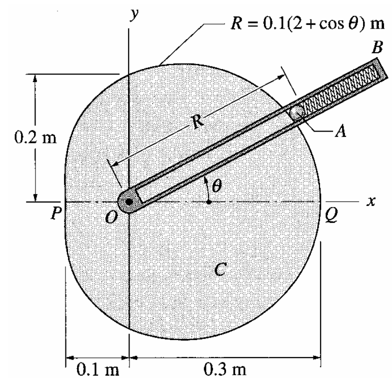
Now we know the components of the normal force.

$$N_\theta = \frac{F_S}{2\pi} - \frac{mb}{4\pi^2}\ddot{\theta} = 2.292\,\text{lb} - 0.057\,\text{lb} = 2.23\,\text{lb}$$

$$N_R = -2\pi N_\theta = -14.0\,\text{lb}$$

$$N = \sqrt{N_R^2 + N_\theta^2} = 14.2\,\text{lb}$$

3 Problem 13.75 (Scott)



The arm OB of the system described in Prob 13.74 ($m_A = 0.10\,\text{kg}$, no friction) rotates with the constant speed $\dot{\theta} = 20\,\text{rad/s}$. When $\theta = 60°$, the force exerted by the spring on the ball A is 12.5 N. For this position, determine the contact force between (a) the ball and the cam; and (b) the ball and the slot.

Solution: The radius of the cam is specified in polar coordinates, so use those. By the description, we already know

$$\theta = 60° \qquad \dot{\theta} = 20\,\text{rad/s} \qquad \ddot{\theta} = 0$$

And by using the formula for R at the current value of θ, we get

$$R = 0.1\,(2 + \cos\theta) = 0.1\,(2 + 0.5) = 0.25\,\text{m}$$

$$\dot{R} = 0.1\left((-\sin\theta)\,\dot{\theta}\right) = 0.1\left(\left(-\sqrt{3}/2\right)20\right) = -1.732\,\text{m/s}$$

$$\ddot{R} = -0.1\left((\sin\theta)\,\ddot{\theta} + (\cos\theta)\,\dot{\theta}^2\right) = -0.1\left(0.5\,(20)^2\right) = -20\,\text{m/s}^2$$

The components of the acceleration are

$$a_R = \ddot{R} - R\dot{\theta}^2 = -20 - 0.25\,(20)^2 = -120\,\text{m/s}^2 \qquad a_\theta = R\ddot{\theta} + 2\dot{R}\dot{\theta} = 2\,(-1.732)\,(20) = -69.28\,\text{m/s}^2$$

These can be plugged directly into the component form of $\vec{F} = m\vec{a}$. Call the cam force $\vec{N}$, the force of the arm $\vec{F}$ (θ direction only), and the force of the spring $\vec{S}$ (R direction only). What we know so far is:

$$\sum F_R = N_R - S = ma_R = (0.1\,\text{kg})\left(-120\,\text{m/s}^2\right) = -12\,\text{N}$$

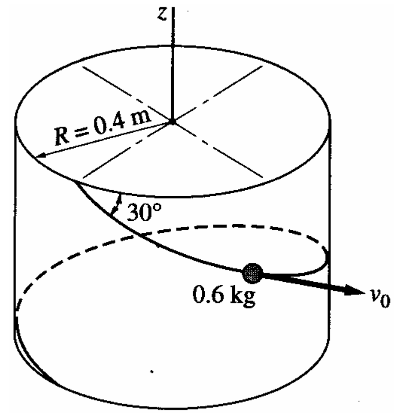
$$\sum F_\theta = N_\theta + F = (0.1\,\text{kg})\left(-69.28\,\text{m/s}^2\right) = -6.928\,\text{N}$$

There are two equations and three unknowns (recall $S = 12.5\,\text{N}$). The missing equation comes from the constraint; $\vec{N}$ is perpendicular to the surface of the cam. We'll use $\vec{v}$ for the direction of the cam.

$$0 = \vec{N}\cdot\vec{v} = (N_R\hat{e}_R + N_\theta\hat{e}_\theta)\cdot\left(\dot{R}\,\hat{e}_R + R\dot{\theta}\,\hat{e}_\theta\right) = -1.732N_R + 5N_\theta = 0$$

Thus, $N_R = 0.5\,\text{N}$, $N_\theta = 0.1732\,\text{N}$, and the magnitude of the cam force is $\boxed{N = 0.529\,\text{N}}$, while $\boxed{F = -7.10\,\text{N}}$. It's negative because it's in the negative $\hat{e}_\theta$ direction while we put it in the $\sum F_\theta$ equation as positive.

4 Problem 13.78 (Matt)



A 0.6 kg particle slides down a rough helical wire at the constant speed $v_0 = 2\,\text{m/s}$. (a) Determine the cylindrical components of the force exerted by the wire on the particle; and (b) find the coefficient of kinetic friction between the particle and the wire.

Solution: Using cylindrical coordinates, the velocity is $\vec{v} = \dot{R}\hat{e}_R + R\dot{\theta}\hat{e}_\theta + \dot{z}\hat{k} = (2\,\text{m/s})\cos 30^\circ \hat{e}_\theta - (2\,\text{m/s})\sin 30^\circ \hat{k} = \left(1.732\hat{e}_\theta - 1\hat{k}\right)\,\text{m/s}$. This means that $\dot{\theta} = v_\theta/R = 1.732/0.4 = 4.330\,\text{s}^{-1}$. The acceleration is $\vec{a} = \left(\ddot{R} - R\dot{\theta}^2\right)\hat{e}_R + \left(R\ddot{\theta} + 2\dot{R}\dot{\theta}\right)\hat{e}_\theta + \ddot{z}\hat{k} = -R\dot{\theta}^2\hat{e}_R = -7.500\hat{e}_R\,\text{m/s}^2$.

For Newton's Second Law, there are 2 forces, gravity and the force of the wire, $\vec{F}$.

$$\sum F_R = F_R = ma_R = (0.6\,\text{kg})\left(-7.5\,\text{m/s}^2\right) = \boxed{-4.5\,\text{N}}$$

$$\sum F_\theta = F_\theta = 0$$

$$\sum F_z = F_z - mg = 0 \quad \rightarrow \quad F_z = mg = (0.6\,\text{kg})\left(9.81\,\text{m/s}^2\right) = \boxed{5.886\,\text{N}}$$

The harder part is to find the normal and tangential components of the force of the wire. The straightforward way is to first find the tangential component by dotting the force with the velocity direction.

$$F_T = \vec{F} \cdot \hat{v} = \frac{\left(-4.5\hat{e}_R + 5.886\hat{k}\right) \cdot \left(1.732\hat{e}_\theta - 1\hat{k}\right)}{\sqrt{1.732^2 + 1^2}} = \frac{5.886}{2} = 2.943\,\text{N}$$

Then, the normal component can be found by the pythagorean theorem

$$F_R^2 + F_\theta^2 + F_z^2 = 54.89\,\text{N}^2 = F_T^2 + F_N^2 = 2.943^2 + F_N^2 \quad \rightarrow \quad F_N = 6.80\,\text{N}$$

The coefficient of friction is

$$\mu = F_T/F_N = 2.943/6.80 = \boxed{0.433}$$