Phys202 - Dynamics - Spring 2007 HW4 Solutions

1 Problem 13.18

A particle moves with a constant speed v_0 along the curve $y = Ax^2+Bx+C$. Find the maximum acceleration and the corresponding x-coordinate.

Solution: The maximum acceleration occurs at the tip of the parabola. This is at the value $x = \frac{-B}{2A}$. The actual curvature is

$$\rho = \frac{\left(1+\left(\frac{dy}{dx}\right)^2\right)^{3/2}}{\left|\frac{d^2y}{dx^2}\right|} = \frac{\left(1+(2Ax+B)^2\right)^{3/2}}{2\left|A\right|} = \frac{\left(1+\left(2A\left(\frac{-B}{2A}\right)+B\right)^2\right)^{3/2}}{2\left|A\right|} = \frac{1}{2\left|A\right|}$$

The acceleration at this point is $a = \frac{v_0^2}{\rho} = \boxed{2\left|A\right| v_0^2}$

2 Problem 13.29

The collar B slides along a guide rod that has the shape of the spiral $R = b\theta$. A pin on the collar slides in the slotted arm OC. If OC is rotating at the constant angular speed $\dot{\theta} = \omega$, determine the magnitude of the acceleration of the collar when it is at A.

Solution: Since R is a function of θ, polar coordinates are appropriate. Note that $\dot{R} = b\dot{\theta} = b\omega$. So,

$$\begin{aligned} \vec{a} &= \left(\ddot{R} - R\dot{\theta}^2\right)\hat{e}_R + \left(R\ddot{\theta} + 2\dot{R}\dot{\theta}\right)\hat{e}_\theta \\ &= \left(-b\theta\omega^2\right)\hat{e}_R + \left(2b\omega^2\right)\hat{e}_\theta \end{aligned}$$

$$a = \sqrt{b^2\theta^2\omega^4 + 4b^2\omega^4} = b\omega^2\sqrt{\theta^2+4}$$

At the point requested, $\theta = \frac{\pi}{2}$, so at A, $\boxed{a = 2.543\, b\omega^2}$.

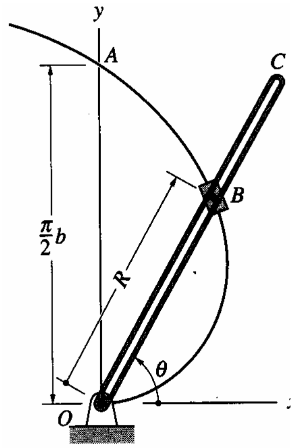

Figure 1: Slider Mechanism for Problem 13.29.

3 Problem 13.34

The curved portion of a cloverleaf highway intercharge is defined by $R^2 = b^2\sin 2\theta$, for $0 \le \theta \le 90°$. If a car travels along the curve at the constant speed v_0, determine its acceleration at A.

Solution: Again, because we have $R(\theta)$, polar coordinates work well. Some preliminary things we need are the derivatives of R. It's slightly easier if we differentiate without taking the square root of both sides. $R^2 = b^2\sin 2\theta$ becomes $2R\dot{R} = 2b^2\dot{\theta}\cos 2\theta$ and the second derivative is $2R\ddot{R} + 2\dot{R}^2 = 2b^2\left(\dot{\theta}^2(-2)\sin 2\theta + \ddot{\theta}\cos 2\theta\right)$. At point A, $\theta = \pi/4$, $\sin 2\theta = 1$, $\cos 2\theta = 0$,

$$\begin{aligned} R &= b \\ \dot{R} &= 0 \\ \ddot{R} &= \frac{-4b^2\dot{\theta}^2}{2b} = -2b\dot{\theta}^2 \end{aligned}$$

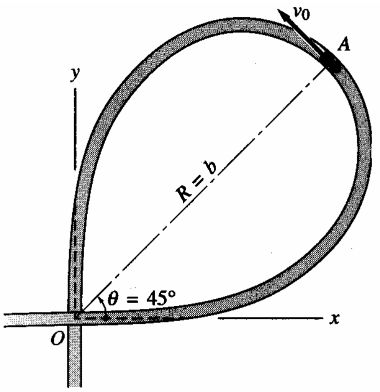

Figure 2: Cloverleaf of Problem 13.34.

The velocity at A is $\vec{v} = \dot{R}\hat{e}_R + R\dot{\theta}\hat{e}_\theta$, so the speed is $v = R\dot{\theta} = b\dot{\theta}$.

The full speed is $v^2 = \dot{R}^2 + R^2\dot{\theta}^2$ and its derivative is $2v\dot{v} = 2\dot{R}\ddot{R} + 2R\dot{R}\dot{\theta}^2 + R^2 2\dot{\theta}\ddot{\theta}$. $\dot{v} = 0$, and at point A, $\dot{R} = 0$, so $\ddot{\theta} = 0$ as well.

The circumferencial acceleration is

$$a_\theta = R\ddot{\theta} + 2\dot{R}\dot{\theta} = 0$$

The radial acceleration component is

$$a_R = \ddot{R} - R\dot{\theta}^2 = -2b\left(\frac{v}{b}\right)^2 - b\left(\frac{v}{b}\right)^2 = \boxed{\frac{-3v^2}{b}}$$

4 Problem 13.36

A helicoptor is tracked by radar, which records R, θ, and $\dot{\theta}$ at regular intervals. The readings at a certain instant are $R = 8050\,\text{ft}$, $\theta = 38.4°$, and $\dot{\theta} = 0.0367\,\text{rad/s}$. If the helicoptor is in level flight, calculate the elevation h and the speed v of the helicoptor at that instant.

Solution: Convert from polar to rectangular coordinates: $x = R\sin\theta$ and $y = R\cos\theta$. Then $h = (8050\,\text{ft})\sin 38.4° = \boxed{5000\,\text{ft}}$.

The velocity, in polar coordinates, is $\vec{v} = v_R\hat{e}_R + v_\theta\hat{e}_\theta = \dot{R}\hat{e}_R + R\dot{\theta}\hat{e}_\theta$. But we don't know $\dot{R}$. Since we do know the velocity is in the x direction, we can convert the velocity back to rectangular coordinates by using the equations for the unit vectors.

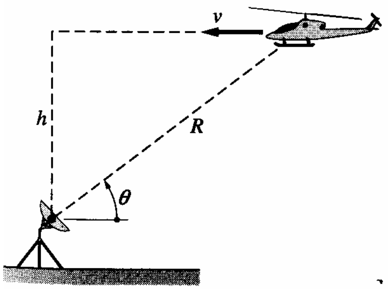

Figure 3: A helicoptor tracked by radar for Problem 13.36.

$$\vec{v} = v_R\hat{e}_R + v_\theta\hat{e}_\theta = v_R\left(\cos\theta\hat{\imath} + \sin\theta\hat{\jmath}\right) + v_\theta\left(-\sin\theta\hat{\imath} + \cos\theta\hat{\jmath}\right) = v_x\hat{\imath} + v_y\hat{\jmath}$$

We do "know" that the y velocity is zero.

$$\begin{aligned} v_y &= v_R\sin\theta + v_\theta\cos\theta = \dot{R}\sin\theta + R\dot{\theta}\cos\theta = 0 \\ \dot{R} &= \frac{-R\dot{\theta}\cos\theta}{\sin\theta} \end{aligned}$$

Now we can calculate the velocity.

$$\begin{aligned} v_x &= v_R\cos\theta - v_\theta\sin\theta = \frac{-R\dot{\theta}\cos^2\theta}{\sin\theta} - R\dot{\theta}\sin\theta \\ &= \frac{-\,(8050\,\text{ft})\left(0.0367\,\text{s}^{-1}\right)\cos^2 38.4°}{\sin 38.4°} - (8050\,\text{ft})\left(0.0367\,\text{s}^{-1}\right)\sin 38.4° \\ &= \boxed{-476\,\text{ft/s}} \end{aligned}$$

Notice that inside the trig functions, we can use degrees if we want. But $\dot{\theta}$ must be in rad/s (s^{-1} is just another way of writing rad/s).

5 Problem 13.54

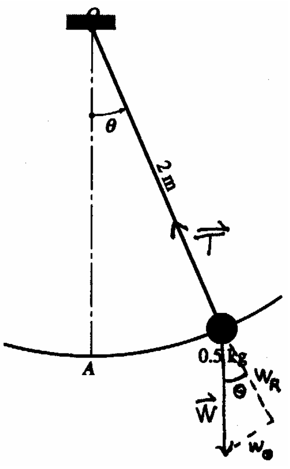

Figure 4: Pendulum with the force diagram superimposed on the drawing.

A 0.5 kg pendulum on a 2 m string is released from rest with $\theta = 30°$.
(a) Derive the equation of motion using θ as the independent variable.
(b) Determine the speed of the bob as a function of θ.

Solution: In polar coordinates, the acceleration components are

$$a_R = \ddot{R} - R\dot{\theta}^2 \qquad a_\theta = R\ddot{\theta} + 2\dot{R}\dot{\theta}$$

The radial component of Newton's Second Law is $\sum F_R = W_R - T = W\cos\theta - T = -mR\dot{\theta}^2$ which can tell us the tension in the string, but not much about the motion. The circumferential equation is more useful

$$\begin{aligned} \sum F_\theta &= -W_\theta = -W\sin\theta = mR\ddot{\theta} \\ \ddot{\theta} &= -\frac{g}{R}\sin\theta \end{aligned}$$

This is the "equation of motion". It can be integrated to get $\dot{\theta}$ (and hence v) as a function of θ.

$$\begin{aligned} \ddot{\theta} &= \frac{d\dot{\theta}}{dt} = \frac{d\dot{\theta}}{d\theta}\dot{\theta} = \boxed{-\frac{g}{R}\sin\theta = \ddot{\theta}} \\ \dot{\theta}\,d\dot{\theta} &= -\frac{g}{R}\sin\theta\,d\theta \\ \frac{1}{2}\dot{\theta}^2 &= \frac{g}{R}\cos\theta + C \end{aligned}$$

The constant C is set by the initial conditions of $\dot{\theta} = 0$ when $\theta = 30°$, so $\cos\theta = \frac{\sqrt{3}}{2}$, and $C = \frac{g\sqrt{3}}{2R}$, and $\dot{\theta} = \sqrt{\frac{g}{R}\left(2\cos\theta - \sqrt{3}\right)}$

So the speed is $\boxed{v = R\dot{\theta} = \sqrt{gR\left(2\cos\theta - \sqrt{3}\right)}}$.

6 Problem 13.98

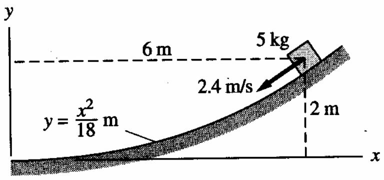

Figure 5: Problem 13.98.

A 5 kg package is sliding down a parabolic chute. In the position shown, the speed of the package is 2.4 m/s. Determine the normal contact force between the chute and the package in this position.

Solution: At the point described, $x = 6\,\text{m}$, $y = \frac{x^2}{18} = 2\,\text{m}$, $\frac{dy}{dx} = \frac{x}{9} = \frac{2}{3}$, and $\frac{d^2y}{dx^2} = \frac{1}{9}\,\text{m}^{-1}$. The curvature is

$$\rho = \frac{\left(1 - \left(\frac{dy}{dx}\right)^2\right)^{3/2}}{\left|\frac{d^2y}{dx^2}\right|} = \frac{\left(1 - (2/3)^2\right)^{3/2}}{1/9} = 15.62\,\text{m}$$

So, the acceleration in the normal direction is $a_n = v^2/\rho = 2.4^2/15.62 = 0.3688\,\text{m/s}^2$. Newton's Second Law in that direction is

$$\sum F_n = N - W\cos\theta = N - (5\,\text{kg})\left(9.81\,\text{m/s}^2\right)\left(\frac{1}{\sqrt{1 + (2/3)^2}}\right) = N - (40.81\,\text{N}) = (5\,\text{kg})\left(0.3688\,\text{m/s}^2\right)$$

So the normal force is $\boxed{N = 42.7\,\text{N}}$.

7 Problem 13.102

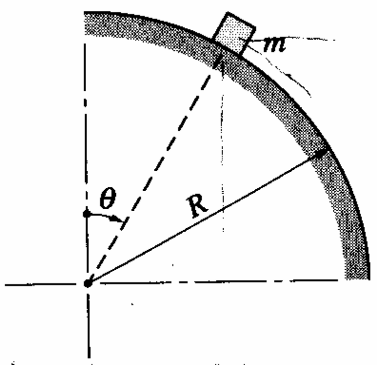

Figure 6:

A small block of mass m slides without friction on a cylindrical surface of radius R. If the block is released from rest at $\theta = 0$, determine the contact force between the block and the surface in terms of θ.

Solution: In circular motion, the acceleration components are

$$a_R = -R\dot{\theta}^2 \qquad a_\theta = R\ddot{\theta}$$

And Newton's Second Law yields

$$\sum F_R = N - W\cos\theta = -mR\dot{\theta}^2 \qquad \sum F_\theta = W\sin\theta = mR\ddot{\theta}$$

We integrate the θ equation

$$\begin{aligned} \frac{g}{R}\sin\theta &= \ddot{\theta} = \frac{d\dot{\theta}}{d\theta}\dot{\theta} \\ \frac{g}{R}\sin\theta\,\mathrm{d}\theta &= \dot{\theta}\,d\dot{\theta} \\ -\frac{g}{R}\cos\theta &= \frac{1}{2}\dot{\theta}^2 + C \end{aligned}$$

When $\theta = 0$, $\dot{\theta} = 0$, so $C = -g/R$ and $\dot{\theta}^2 = \frac{2g}{R}(1-\cos\theta)$. Plug this into the radial $F = ma$ to get

$$\begin{aligned} N - mg\cos\theta &= -mR\left(\frac{2g}{R}(1-\cos\theta)\right) \\ N &= mg\cos\theta - mR\left(\frac{2g}{R}(1-\cos\theta)\right) \\ N &= \boxed{mg\,(3\cos\theta - 2)} \end{aligned}$$

Note that a negative force isn't possible, so $\cos\theta > \frac{2}{3}$ or $\theta < 48.2°$.