

1 Problem 12.41

Solve Prob. 12.40 if the propulsive force decays exponentially according to  $F(t) = 1000e^{-0.2t}$ , where  $F(t)$  is in pounds and  $t$  is the time in seconds measured from the instant the rocket engine was fired. (12.40: A 3000-lb rocket sled is propelled along a straight test track. The rocket engine fires for 4 seconds, producing a propulsive force of  $F = 1200$  lb, and then shuts down. Assuming that the sled starts from rest and that the coefficient of kinetic friction is 0.05, determine the maximum speed reached by the sled.)

**Solution:** This is a 1-D problem because the rocket only goes straight along the ground, so  $a = \dot{v} = \ddot{x}$ . The mass is  $m = W/g = (3000 \text{ lb}) / (32.2 \text{ ft/s}^2) = 93.17 \text{ slug}$ . The friction force is  $f = \mu_k N = \mu_k W = (0.05)(3000 \text{ lb}) = 150 \text{ lb}$ . The acceleration is found from Newton's Second Law:

$$\begin{aligned} \sum F_x = F(t) - f = 1000e^{-0.2t} - 150 &= ma = m \frac{dv}{dt} \\ (1000e^{-0.2t} - 150) dt &= m dv \\ -5000e^{-0.2t} - 150t &= 93.17v + C \end{aligned}$$

To find  $C$ , set  $v = 0$  at  $t = 0$ . So, under this force, the velocity is

$$\begin{aligned} v(t) &= 53.67(1 - e^{-0.2t}) - 1.610t \\ v(4) &= 29.6 \text{ ft/s} \end{aligned}$$

To make sure this is really the maximum velocity, we should check to make sure the object is actually accelerating (as opposed to decelerating). Setting  $F(t) - f = 0$  yields:

$$\begin{aligned} (1000 \text{ lb}) e^{-0.2t} &= 150 \text{ lb} \\ e^{-0.2t} &= 150/1000 \\ -0.2t &= \ln(0.15) \\ t &= 9.846 \text{ s} \end{aligned}$$

This is the time when the friction starts to overcome the thrust, so when the engine shut off, the rocket really was at its max speed so far.

2 Problem 12.45

The drag force acting on a 135-lb skydiver in the "spread" position can be approximated by  $F_D = 0.00436v^2$ , where  $F_D$  is in pounds and  $v$  is in feet per second. Assuming that the skydiver follows a vertical path, determine the terminal velocity.

**Solution:** Terminal velocity occurs when  $a = 0$ , so

$$\begin{aligned} F_D &= W \\ 0.00436v^2 &= 135 \\ v &= \sqrt{135/0.00436} = 176 \text{ ft/s} = 120 \text{ MPH} \end{aligned}$$

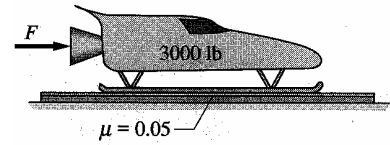


Fig. P12.40, P12.41

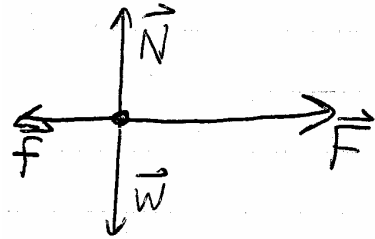


Figure 1: Sketch and Force Diagram for problem 12.41.

### 3 Problem 12.55

A car of mass  $m$  travels along a cloverleaf interchange. The position of the car is given by

$$x = \frac{b}{2} \left( \sin \frac{\pi t}{4t_0} + \sin \frac{3\pi t}{4t_0} \right) \quad y = \frac{b}{2} \left( \cos \frac{\pi t}{4t_0} - \cos \frac{3\pi t}{4t_0} \right)$$

where  $b = 240$  m, and  $t_0 = 12$  s is the time of travel between  $O$  and  $A$ . Determine the smallest coefficient of friction between the tires and the road that would prevent the car from skidding at  $A$ .

**Solution:** Since rectangular coordinates are used,  $v_x = \dot{x}$  and  $a_x = \dot{v}_x$  (same for  $y$ ).

$$\dot{x} = \frac{b}{2} \frac{\pi}{4t_0} \left( \cos \frac{\pi t}{4t_0} - 3 \cos \frac{3\pi t}{4t_0} \right) \quad \dot{y} = \frac{b}{2} \frac{\pi}{4t_0} \left( -\sin \frac{\pi t}{4t_0} + \sin \frac{3\pi t}{4t_0} \right)$$

The acceleration is

$$a_x = -\frac{b}{2} \left( \frac{\pi}{4t_0} \right)^2 \left( \sin \frac{\pi t}{4t_0} + 9 \sin \frac{3\pi t}{4t_0} \right) \quad a_y = -\frac{b}{2} \left( \frac{\pi}{4t_0} \right)^2 \left( \cos \frac{\pi t}{4t_0} - \cos \frac{3\pi t}{4t_0} \right)$$

$$\begin{aligned} a_x(t=t_0) &= -\frac{b}{2} \left( \frac{\pi}{4t_0} \right)^2 \left( \frac{\sqrt{2}}{2} + 9 \frac{\sqrt{2}}{2} \right) \\ &= -3.635 \text{ m/s}^2 \end{aligned}$$

At  $t = t_0$ ,  $a_y = a_x = -3.635 \text{ m/s}^2$ , are the components, so the magnitude of the acceleration is  $a = \sqrt{2} \cdot 3.635 = 5.14 \text{ m/s}^2$ .

Now to analyze the kinetics. The normal force is equal to the weight, which is  $mg$ , so the maximum friction force is  $\mu_k mg$ . This means the maximum acceleration due to the friction is  $\mu_k g$ . Setting this equal to the calculated acceleration gives the minimum coefficient of friction

$$\mu_k = \frac{a_{\max}}{g} = \frac{5.14 \text{ m/s}^2}{9.81 \text{ m/s}^2} = 0.524$$

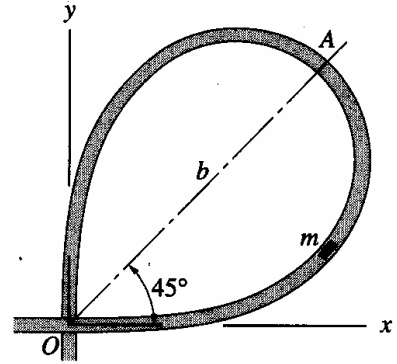


Figure 2: Cloverleaf interchange for Problem 12.55.

## 4 Problem 12.58

A projectile, launched at  $A$  with an initial velocity of 80 ft/s at the angle  $\theta$ , impacts the wall at  $B$ . Compute the angle  $\theta$  that will maximize the height  $h$  of the impact point. What is this maximum height?

**Solution:** This is 2-D motion best analyzed in rectangular coordinates. With only the force of gravity, the equations of projectile motion are

$$x = v_{0x}t \quad y = v_{0y}t - \frac{1}{2}gt^2$$

The initial velocity components are

$$v_{0x} = v_0 \cos \theta \quad v_{0y} = v_0 \sin \theta$$

Solve for  $t$  at some final  $x = x_f$  and plug into the  $y$  equation to find the final height  $h$ .

$$t = \frac{x_f}{v_0 \cos \theta} \quad h = \frac{v_0 \sin \theta x_f}{v_0 \cos \theta} - \frac{1gx_f^2}{2v_0^2 \cos^2 \theta} = x_f \tan \theta - \frac{gx_f^2}{2v_0^2 \cos^2 \theta}$$

Now maximize  $h$  by adjusting  $\theta$

$$\begin{aligned} \frac{dh}{d\theta} = 0 &= \frac{x_f}{\cos^2 \theta} - \frac{gx_f^2}{2v_0^2} \left( \frac{2 \sin \theta}{\cos^3 \theta} \right) \\ 0 &= x_f - \frac{gx_f^2}{v_0^2} \tan \theta \\ \tan \theta &= \frac{v_0^2}{gx_f} = \frac{(80 \text{ ft/s})^2}{(32.2 \text{ ft/s}^2)(60 \text{ ft})} = 3.313 \\ \theta &= 73.2^\circ \end{aligned}$$

The final height is

$$h = (60 \text{ ft})(3.313) - \frac{(32.2 \text{ ft/s}^2)(60 \text{ ft})^2}{2(80 \text{ ft/s})^2} \frac{1}{(0.2890)^2} = 90.3 \text{ ft}$$

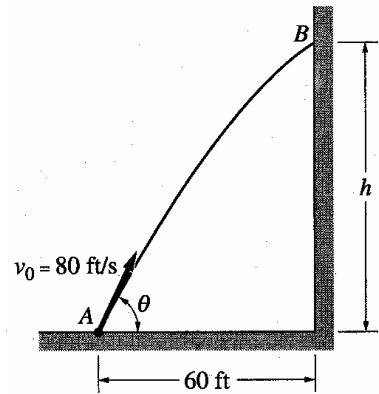


Figure 3: Problem 12.58.

## 5 Problem 12.108

During braking, the speed of a car traveling along a straight highway varies as  $v = 12 - (x/4)$  m/s, where  $x$  (in meters) is the distance traveled after the brakes are applied. Determine the acceleration of the car as a function of  $x$ .

**Solution:** In this 1-D problem, the acceleration is simply

$$a = \frac{dv}{dt} = \frac{dv}{dx} \frac{dx}{dt} = v \cdot \frac{d}{dx} \left( 12 - \frac{x}{4} \right) = -\frac{v}{4} = -\frac{1}{4} \left( 12 - \frac{x}{4} \right) = \frac{x}{16} - 3 \text{ m/s}^2$$

## 6 Problem 12.109

For the car described in Problem 12.108 ( $v = 12 - x/4$ ), determine  $x$  as a function of time, assuming braking begins at  $t = 0$ .

**Solution:** This is a first-order linear differential equation for  $x$ , so we have to integrate the velocity, using  $x(0) = 0$  to set the constant.

$$\begin{aligned} \frac{dx}{dt} &= 12 - \frac{x}{4} \\ \frac{dx}{12 - \frac{x}{4}} &= dt \\ \ln \left( 12 - \frac{x}{4} \right) (-4) &= t + C_1 \\ \ln \left( 12 - \frac{x}{4} \right) &= \frac{-t}{4} + C_2 \\ 12 - \frac{x}{4} &= C_3 e^{-t/4} \\ \frac{-x}{4} &= 12e^{-t/4} - 12 \\ x &= 48 \left( 1 - e^{-t/4} \right) \end{aligned}$$

## 7 Prof. Larrabee's Problem

If an object starts from rest and accelerates at  $g$ , how far would it travel before it reached the speed of light, ignoring special relativity?

**Solution:** The easiest way to do this is by conservation of energy.

$$\begin{aligned} \frac{1}{2}mv^2 &= mgx \\ \frac{v^2}{2g} &= x = \frac{(3 \times 10^8 \text{ m/s})^2}{2 \cdot (9.81 \text{ m/s}^2)} = 441 \times 10^{15} \text{ m} = 2.95 \times 10^9 \text{ A.U.} = 46.7 \text{ ly} \end{aligned}$$

To go 46.7 light-years at an average speed of half the speed of light would take 93.3 years!