Phys202 - Dynamics - Spring 2007 HW1 Solutions

1 Problem 11.1

A person weighs 30 lb on the moon, where $g = 5.32\,\mathrm{ft/s^2}$. Determine (a) the mass of the person; and (b) the weight of the person on earth.

Solution: Weight an mass are related by $W = mg$, so

$$m = \frac{W}{g} = \frac{30\,\mathrm{lb}}{5.32\,\mathrm{ft/s^2}} = 5.64\,\mathrm{lb \cdot s^2/ft} = 5.64\,\mathrm{slug}$$

It is the mass that doesn't change. Back on earth, $g = 32.3\,\mathrm{ft/s^2}$. $W = mg = (5.64\,\mathrm{slug})\left(32.2\,\mathrm{ft/s^2}\right) = 182\,\mathrm{lb}$.

2 Problem 11.2

The radius and length of a steel cylinder are 60 mm and 120 mm, respectively. If the mass density of steel is 7850 $\mathrm{kg/m^3}$, determine the weight of the cylinder in pounds.

Solution: The volume of a cylinder is $V = \pi r^2 L = \pi\,(0.06\,\mathrm{m})^2\,(0.12\,\mathrm{m}) = 1.357 \times 10^{-3}\,\mathrm{m^3}$. The mass is $m = \rho V = \left(7580\,\mathrm{kg/m^3}\right)\left(1.357 \times 10^{-3}\,\mathrm{m^3}\right) = 10.65\,\mathrm{kg}$. We can convert to slugs first, then find the weight, or find the weight in SI and convert from Newtons to pounds.

$$W = mg = (10.65\,\mathrm{kg})\left(9.81\,\mathrm{m/s^2}\right) = 104.5\,\mathrm{N}\left(\frac{1\,\mathrm{lb}}{4.448\,\mathrm{N}}\right) = 23.49\,\mathrm{lb} \approx 23\,\mathrm{lb}$$

There is no proper direct conversion from kg to lb.

3 Problem 11.14

A famous equation of Einstein is $E = mc^2$, where E is energy, m is mass, and c is the speed of light. Determine the dimension of energy in terms of the base dimensions of (a) a gravitational $[FLT]$ system; and (b) an absolute $[MLT]$ system.

Solution: (a) Mass is weight divided by acceleration:

$$[M] = \left[\frac{FT^2}{L}\right]$$

So, energy must be in units of:

$$[E] = [MS^2] = \left[\frac{FT^2}{L} \cdot \left(\frac{L}{T}\right)^2\right] = [FL]$$

(b) In the SI system, mass is already a base unit. So:

$$[E] = \left[MS^2\right] = \left[M \cdot \left(\frac{L}{T}\right)^2\right] = \left[\frac{ML^2}{T^2}\right]$$

This dimensional analysis doesn't actually tell us how big the unit of energy is as compared to the given product of other units, but in the SI system, it is a simple product.

$$1\,\mathrm{J} = 1\frac{\mathrm{kg \cdot m^2}}{\mathrm{s^2}}$$

4 Problem 11.20

Find the height h (km) above the earth's surface where the gravitational attraction is one-half of its value on earth. (Note that ISS orbits at about 340 km above the surface. Why are people "weightless" on ISS?)

Solution: You can do this by looking at the equation for gravity and thinking about it:

$$F = G\frac{mM}{r^2}$$

With 1/2 of the force, the distance r must have increased by a factor of $\sqrt{2}$. If the situation is too complicated to understand "by inspection", another way is to consider the two cases explicitly, labeling the quantities that change from one situation to the other with subscripts.

$$\begin{aligned} G\frac{mM}{r_2^2} = F_2 &= \frac{1}{2}F_1 = \frac{1}{2}G\frac{mM}{r_1^2} \\ r_2^2 &= 2r_1^2 \\ r_2 &= \sqrt{2}\cdot r_1 \end{aligned}$$

On ISS, both the space station and the people are in constant free-fall, so they experience the same acceleration and the same relative motion.

5 Problem 12.9

A projectile fired from the origin O follows a parabolic trajectory, given in parametric form by

$$x = 86t \qquad y = 96t - 4.91t^2$$

where x and y are measured in meters and t is in seconds. Determine (a) the acceleration vector throughout the flight; (b) the velocity vector at O; (c) the maximum height h; and (d) the range L.

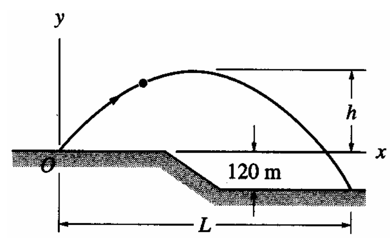

Figure 1: A projectile fired off a 120 m cliff.

Solution: In this 2-D rectangular coordinate system, the components of velocity are (notice the roundoff error in g):

$$v_x = \dot{x} = 86 \qquad v_y = \dot{y} = 96 - 9.82t$$

The components of acceleration are $a_x = \dot{v}_x = 0 \quad a_y = \dot{v}_y = -9.82$, so the total acceleration is $\boxed{\vec{a} = -9.82\hat{\jmath}}$. The components of velocity at O (where $t = 0$) are $v_x = 86\,\mathrm{m/s}$ and $v_y = 96\,\mathrm{m/s}$, so $\boxed{\vec{v} = 86\hat{\imath} + 96\hat{\jmath}\,\mathrm{m/s}}$, while $v = \sqrt{v_x^2 + v_y^2} = 129\,\mathrm{m/s}$. The maximum height is found from setting $v_y = 0 = 96 - 9.82t$, so $t = 9.78\,\mathrm{s}$ and plugging this back into the position equation, $y(t = 9.78) = 96\,(9.78) - 4.91\,(9.78)^2$ therefore $\boxed{h = 469\,\mathrm{m}}$.

The range is found from setting $y = -120\,\mathrm{m}$ (the final height), solving for t, then finding x. $96t - 4.91t^2 = -120$ or $4.91t^2 - 96t - 120 = 0$, so $t = \frac{96\pm\sqrt{96^2-4(4.91)(-120)}}{2(4.91)} = 9.776 \pm 10.955\,\mathrm{s}$. We want the positive root, so $t = 20.731\,\mathrm{s}$. Then $\boxed{x = 86t = 1780\,\mathrm{m}}$.