Phys202 - Dynamics - Spring 2007 HW19 Solutions

HW19 - Fri, May 4 - Prove $A\cos(pt) + B\sin(pt) = E\sin(pt+\alpha)$. Find A and B in terms of E and α. Problems 20.1, 20.8, 20.29, 20.38, 20.84.

20.1

(a) $x(t) = E\sin(pt+\alpha)$

$$p = \sqrt{k/m} = \sqrt{500/20} = 5 \text{ rad/s}$$

$$E = \sqrt{x_0^2 + (v_0/p)^2} = \sqrt{(0.0455)^2 + (-0.104/5)^2} = 0.0500 \text{ m} \ \blacklozenge$$

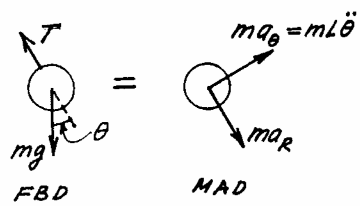
(b) $\alpha = \tan^{-1}(x_0 p/v_0) = \tan^{-1}\left[(0.0455)(5)/(-0.104)\right] = -1.142 \text{ rad}$

$$v(t) = \dot{x}(t) = pE\cos(pt+\alpha)$$

$$v = 0 \text{ when } pt+\alpha = \frac{\pi}{2} \qquad \therefore t = \frac{1}{p}\left(\frac{\pi}{2}-\alpha\right) = \frac{1}{5}\left(\frac{\pi}{2}+1.142\right) = 0.543 \text{ s} \ \blacklozenge$$

20.8

Derive equation of motion first.



$$\Sigma F_\theta = ma_\theta: \ \nearrow\!+ \quad -mg\sin\theta = mL\ddot{\theta}$$

$$\therefore \ddot{\theta} = -\frac{g}{L}\sin\theta \quad \text{or} \quad \ddot{\theta} = -p^2\sin\theta$$

where $p^2 = g/L$

(a) If $\theta << 1$, then $\sin\theta \approx \theta$ and the equation of motion simplifies to $\ddot{\theta} + p^2\theta = 0$

$$\therefore \theta(t) = \theta_0\cos pt \quad \dot{\theta}(t) = -p\theta_0\sin pt \quad \ddot{\theta}(t) = -p^2\theta_0\cos pt$$

$$\therefore \dot{\theta}_{max} = p\theta_0 \ \blacklozenge \qquad \ddot{\theta}_{max} = p^2\theta_0 \ \blacklozenge$$

20.8 continued

(b) Equation of motion can be written as $\frac{d\dot{\theta}}{d\theta}\dot{\theta} = -p^2\sin\theta$

$$\therefore \dot{\theta}\,d\dot{\theta} = -p^2\sin\theta\,d\theta \qquad \therefore \frac{1}{2}\dot{\theta}^2 = p^2(\cos\theta - \cos\theta_0)$$

$$\therefore \dot{\theta} = p\sqrt{2(\cos\theta - \cos\theta_0)} \quad \therefore \dot{\theta}_{max} = p\sqrt{2(1-\cos\theta_0)} \ \blacklozenge$$

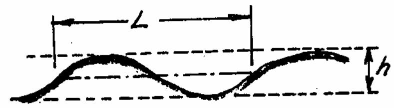
From equation of motion: $\ddot{\theta}_{max} = p^2\sin\theta_0 \ \blacklozenge$

(c)

	Simple harmonic		Exact	
θ (deg)	$\dot{\theta}_{max}/p$	$\ddot{\theta}_{max}/p^2$	$\dot{\theta}_{max}/p$	$\ddot{\theta}_{max}/p^2$
5	0.08727	0.08727	0.08724	0.08716
10	0.17453	0.17453	0.17431	0.17365
15	0.3491	0.3491	0.3473	0.3420

20.29

The amplitude of the relative motion (displacement of the trailer relative to the wheels) of the steady-state vibration is



$$Z = Y\frac{(\omega/p)^2}{1-(\omega/p)^2}$$

where $Y = \frac{h}{2} = \frac{1.5/12}{2} = 0.0625$ ft,

$$\omega = 2\pi\frac{v}{L},\quad p^2 = \frac{2k}{W/g} = \frac{2(240\times 12)}{800/32.2} = 231.8\ (\text{rad/s})^2$$

$\therefore$ The amplitude of the absolute motion is

$$X = Z + Y = Y\left[1+\frac{(\omega/p)^2}{1-(\omega/p)^2}\right] = \frac{Y}{1-(\omega/p)^2}$$

$$\therefore\ x(t) = X\sin\omega t = \frac{Y}{1-(\omega/p)^2}\sin\omega t \quad \therefore\ \ddot{x}(t) = -\frac{Y\omega^2}{1-(\omega/p)^2}\sin\omega t$$

Wheels are about to leave the ground if $|\ddot{x}|_{max} = g$, i.e. $\left|\frac{Y\omega^2}{1-(\omega/p)^2}\right| = g$

<u>If $\omega/p < 1$:</u> $\frac{Y\omega^2}{1-(\omega/p)^2} = g \quad \therefore\ \omega = \sqrt{\frac{g}{Y+g/p^2}} = \sqrt{\frac{32.2}{0.0625+32.2/231.8}} = 12.644$ rad/s

$$\therefore\ v = \frac{\omega L}{2\pi} = \frac{(12.644)(6)}{2\pi} = 12.07\ \text{ft/s}$$

<u>If $\omega/p > 1$:</u> $\frac{Y\omega^2}{(\omega/p)^2-1} = g \quad \therefore\ \omega = \sqrt{\frac{g}{g/p^2-Y}} = \sqrt{\frac{32.2}{32.2/231.8-0.0625}} = 20.53$ rad/s

$$\therefore\ v = \frac{\omega L}{2\pi} = \frac{(20.53)(6)}{2\pi} = 19.60\ \text{ft/s}$$

$\therefore$ Wheels do not stay on ground if $12.07\ \text{ft/s} < v < 19.60\ \text{ft/s}$ ♦

20.38

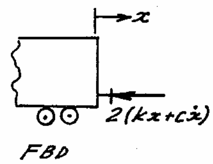
$\Sigma_x F = ma_x$: $\overset{+}{\rightarrow}$ $-2kx - 2c\dot{x} = m\ddot{x}$

Comparing with the "standard" equation

$m\ddot{x} + c\dot{x} + kx = 0$, we conclude that

$$p = \sqrt{\frac{2k}{m}} = \sqrt{\frac{2(12 \times 10^3)}{(180 \times 10^3/32.2)}} = 2.072 \text{ rad/s}$$

$$\zeta = \frac{2c}{2mp} = \frac{2(45 \times 10^3)}{2(180 \times 10^3/32.2)(2.072)} = 3.885$$



(continued)

20.38 continued

System is overdamped ($\zeta > 1$). Therefore, its motion is described by

$$x(t) = A_1 \exp\left[\left(-\zeta + \sqrt{\zeta^2 - 1}\right)pt\right] + A_2 \exp\left[\left(-\zeta - \sqrt{\zeta^2 - 1}\right)pt\right]$$

$$\left(-\zeta \pm \sqrt{\zeta^2 - 1}\right)p = \left(-3.885 \pm \sqrt{3.885^2 - 1}\right)(2.072) = \begin{cases} -0.271 \text{ rad/s} \\ -15.829 \text{ rad/s} \end{cases}$$

$$\therefore x = A_1 \exp(-0.271t) + A_2 \exp(-15.829t)$$

Initial conditions:

$x = 0$ at $t = 0$ $\quad \therefore A_1 + A_2 = 0 \quad \therefore A_2 = -A_1$

$\dot{x} = v_0$ at $t = 0$ $\quad \therefore -0.271A_1 - 15.829A_2 = v_0 \quad \therefore 15.558A_1 = v_0$

$$\therefore A_1 = -A_2 = \frac{v_0}{15.558} = 0.06428v_0$$

$$\therefore x = 0.06428v_0\left[\exp(-0.271t) - \exp(-15.829t)\right]$$

$$\therefore \dot{x} = -0.01742v_0 \exp(-0.271t) + 1.0175v_0 \exp(-15.829t)$$

$\dot{x} = 0$ when $x = x_{max}$. $\quad \therefore -0.01742 \exp(-0.271t) + 1.0175 \exp(-15.829t) = 0$

$$\therefore \frac{\exp(-0.271t)}{\exp(-15.829t)} = \frac{1.0175}{0.01742} \quad \therefore \exp(15.558t) = 58.41 \quad \therefore t = \frac{\ln 58.41}{15.558} = 0.2614 \text{ s}$$

$x = 1.0$ ft when $t = 0.2614$ s.

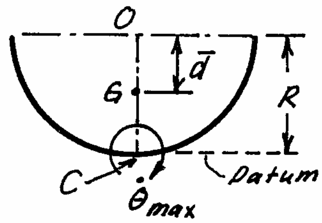
$$\therefore 0.06428\, v_0\left[\exp(-0.271 \times 0.2614) - \exp(-15.829 \times 0.2614)\right] = 1.0$$

$\therefore v_0 = 16.99$ ft/s ♦

20.84

Use Rayleigh's principle.

Position of T_{max}:



$$V_0 = mg(R - \bar{d}) = mg\left(R - \frac{2}{\pi}R\right) = 0.3634mgR$$

$$T_{max} = \frac{1}{2}I_C\dot{\theta}^2_{max} = \frac{1}{2}I_C(p\theta_{max})^2$$

$$I_C = \bar{I} + m(R - \bar{d})^2 = (I_O - m\bar{d}^2) + m(R - \bar{d})^2$$

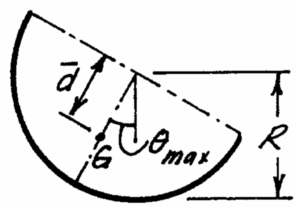
$$= I_O + mR(R - 2\bar{d}) = mR^2 + mR^2\left(1 - 2\frac{2}{\pi}\right) = 0.7268\ mR^2$$

$$\therefore T_{max} = 0.3634\ mR^2p^2\theta^2_{max}$$

(continued)

20.84 continued

Position of V_{max}:



$$V_{max} = mg(R - \bar{d}\cos\theta_{max}) \approx mg\left[R - \bar{d}\left(1 - \frac{1}{2}\theta^2_{max}\right)\right]$$

$$= mgR\left[1 - \frac{2}{\pi}\left(1 - \frac{1}{2}\theta^2_{max}\right)\right]$$

$$= mgR\ (0.3634 + 0.3183\ \theta^2_{max})$$

$$T_{max} = V_{max} - V_0:\ \ 0.3634\ mR^2p^2\theta^2_{max} = 0.3183\ mgR\theta^2_{max}$$

$$\therefore p = \sqrt{\frac{0.3183\ g}{0.3634\ R}} = 0.936\sqrt{\frac{g}{R}}\ \blacklozenge$$