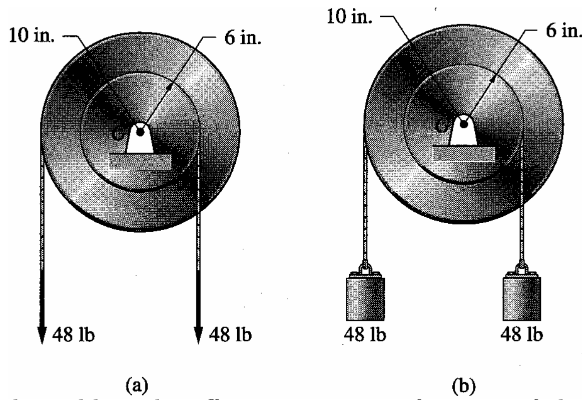
Phys202 - Dynamics - Spring 2007 HW18 Solutions

Problems 17.30 (Just explain which one is larger and why), 18.24, 18.46, 18.64, 18.72, 18.84

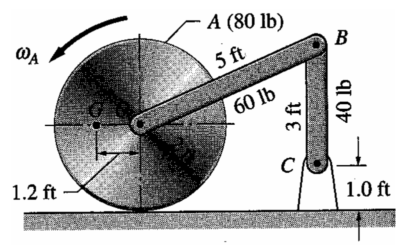
1 Problem 17.30 (Modified)



Determine which of the accelerations of the homogeneous pulleys in figures (a) and (b) is larger and explain why.

Solution: The tension in the ropes in (a) is 48 lb, but the tension in the ropes in (b) is different because the weights accelerate. In the left weight (which falls), the tension is reduced, and in the right weight (which rises), the tension is increased. Since the left tension has a larger moment arm, the net effect is that the torque on the pulley is decreased. System (a) therefore accelerates faster. Another way of thinking about it is that the weights add to the effective moment of inertia of the system while the net external force (gravity) is the same, thus a lower acceleration.

2 Problem 18.24



The radius of gyration of disc A about its center of mass G is $\overline{k} = 1.4\,\text{ft}$. In the position shown, the disc rolls without slipping with an angular velocity $\omega_A = 4\,\text{rad/s}$. Determine the kinetic energy of the system in this position.

Solution: Add the kinetic energies of each part.

Wheel A: Let point D be the point where the wheel contacts the floor. It is an instant center, so for kinetic energy we can consider rotations about that point. The moment of inertia of disc A about point D is:

$$I_D = \overline{I}_D + md^2 = m_A\overline{k}_A^2 + m_A\left(R_A^2 + R_G^2\right) = \left(\frac{80\,\text{lb}}{32.2\,\text{ft/s}^2}\right)\left((1.4\,\text{ft})^2 + (2\,\text{ft})^2 + (1.2\,\text{ft})^2\right) = 18.385\,\text{slug}\cdot\text{ft}^2$$

Its kinetic energy is then $T_A = \frac{1}{2}I_D\omega_A^2 = \frac{1}{2}\left(18.385\,\text{slug}\cdot\text{ft}^2\right)\left(4\,\text{s}^{-1}\right)^2 = 147.08\,\text{lb}\cdot\text{ft}$.

Bar OB: The two key points are related to the other objects. On the left, $\vec{v}_O = R_A\omega_A(-\hat{\imath}) = -8\,\text{ft/s}\,\hat{\imath}$. On the right, $\vec{v}_B = -(3\,\text{ft})\,\omega_{BC}\hat{\imath}$. The relative velocity equation relates the two ends of the bar.

$$\begin{aligned}\vec{v}_B &= \vec{v}_O + \vec{\omega}_{OB}\otimes\vec{r}_{B/O} \\ -(3\,\text{ft})\,\omega_{BC}\hat{\imath} &= -(8\,\text{ft/s})\,\hat{\imath} + \omega_{OB}r_{B/Ox}\hat{\jmath} - \omega_{OB}r_{B/Oy}\hat{\imath}\end{aligned}$$

There is only one term with $\hat{\jmath}$, so $\omega_{OB} = 0$, $\vec{v}_B = \vec{v}_O$, and $-3\omega_{BC} = -8 \rightarrow \omega_{BC} = 2.667\,\text{s}^{-1}$.

The kinetic energy of the (non-rotating) bar OB is $T_{OB} = \frac{1}{2}m\overline{v}^2 = \frac{1}{2}\left(\frac{60\,\text{lb}}{32.2\,\text{ft/s}^2}\right)(8\,\text{ft/s})^2 = 59.63\,\text{lb}\cdot\text{ft}$.

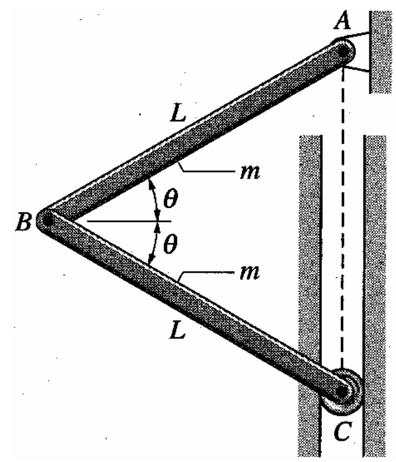
Bar BC: This object is rotating about point C. Its moment of inertia about C is $I_C = \frac{1}{12}mL^2 + m\left(\frac{L}{2}\right)^2 = \frac{1}{3}mL^2 = \frac{1}{3}\left(\frac{40\,\text{lb}}{32.2\,\text{ft/s}^2}\right)(3\,\text{ft})^2 = 3.727\,\text{slug}\cdot\text{ft}^2$. Its kinetic energy is

$$T_{BC} = \frac{1}{2}I_C\omega_{BC}^2 = \frac{1}{2}\left(3.727\,\text{slug}\cdot\text{ft}^2\right)\left(\frac{8}{3}\,\text{s}\right)^2 = 13.25\,\text{lb}\cdot\text{ft}$$

The total kinetic energy is then

$$T = (147.08 + 59.63 + 13.25)\,\text{lb}\cdot\text{ft} = \boxed{219.96\,\text{lb}\cdot\text{ft}}$$

3 Problem 18.46

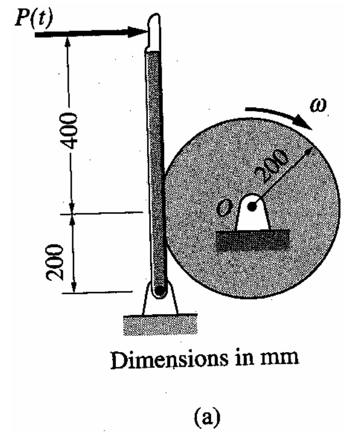


The linkage shown consists of two identical bars, each of length L and mass m. If the linkage is released from rest at $\theta = 0$, find the angular velocity of each link when $\theta = 90°$. Neglect friction.

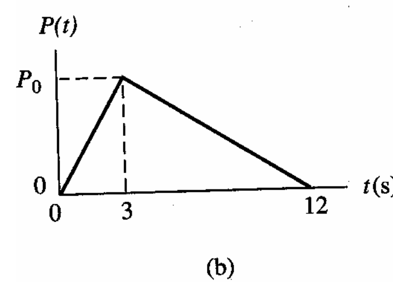
Solution: This is a conservation of energy problem. Assign the datum (zero of height) to be point A. Initially, both bars are at height 0 and nothing is moving, so $E_i = 0$. In the final state, one bar is $\frac{L}{2}$ below A, while the other is $\frac{3L}{2}$ below A. So, $V_f = -2mgL$.

In the final state, wheel C has been moving downwards and stops for an instant before it moves back up. So it is an instant center for bar BC. Each moment of inertia is $I = \frac{1}{3}mL^2$. Point B is rotating about points A and C and is moving to the right, so $\vec{v}_B = v_B\hat{\imath}$ and $v_B = \omega_{AB}L = -\omega_{BC}L$, so the two angular speeds are equal and opposite. The kinetic energy is $T_f = \frac{1}{2}I\omega^2 + \frac{1}{2}I\omega^2 = 2\left(\frac{1}{2}\left(\frac{1}{3}mL^2\right)\omega^2\right)$. So, $2mgL = 2\frac{1}{6}mL^2\omega^2$ or $\omega = \sqrt{6g/L} = \boxed{2.45\sqrt{g/L}}$.

4 Problem 18.64



The disk in part (a) of the figure has a mass of 30 kg and its radius of gyration about O is 172 mm. The disk is spinning freely at $\omega = 500\,\text{rev/min}$ when the force $P(t)$ is applied to the handle of the brake at $t = 0$. The $P - t$ relationship is shown in part (b). Determine the peak value P_0 of the force for which the disc will come to rest at $t = 12\,\text{s}$. The kinetic coefficient of friction between the disc and brake is 0.4.



Solution: The force on the top of the brake lever exerts a normal force on the wheel. The corresponding friction force exerts a torque on the wheel. This torque over a period of 12 s is an angular impulse, which stops the angular momentum of the wheel. The initial angular speed is $\omega = (2\pi/60)(600) = 52.36\,s^{-1}$.

The angular momentum is

$$
\begin{aligned}
h_O &= I_O\omega = mk^2\omega \\
&= (30\,\text{kg})(0.172\,\text{m})^2\left(52.36\,s^{-1}\right) = 46.47\,\text{kg}\cdot\text{m}^2/\text{s}
\end{aligned}
$$

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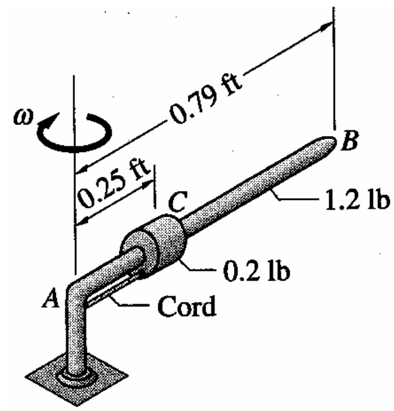
By taking the sum of torques on the bar about the pivot point, the normal force is $N = \frac{600}{200}P(t)$. The friction force is $f = \mu N = 1.2P(t)$. The moment of the friction is $\tau = fR = 1.2\,(0.2\,\text{m})\,P(t) = 0.24P(t)$. The angular impulse is

$$A = \int \tau\,\text{dt} = 0.24\int P(t)\,\text{dt} = 0.24\left(\frac{1}{2}(12)\,P_0\right) = 1.44P_0$$

Setting these equal, $P_0 = 46.47/1.44 = \boxed{32.27\,\text{N}}$.

5 Problem 18.72



The 1.2 lb uniform rod AB and the 0.2 lb small slider C rotate freely about the vertical axis at A. The angular velocity of the system is 5 rad/s when the cord holding C breaks. Determine the angular velocity of AB when (a) C is just about to leave the rod; and (b) just after C has left the rod.

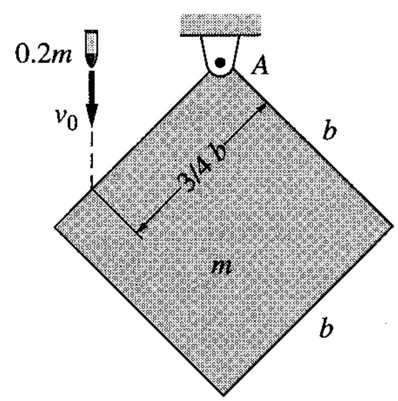
Solution: Since the system is spinning freely, there are no external torques and conservation of angular momentum applies. With a pivot point, the initial and final angular momenta are both $I\omega$. The moment of inertia of the bar is $I_{AB} = \frac{1}{3}m_{AB}L^2$ and the moment of inertia of the slider is $I_C = m_C r_C^2$. $h_{Ai} = h_{Af}$ becomes

$$\left(\frac{1}{3}\left(\frac{1.2\,\text{lb}}{32.2\,\text{ft/s}^2}\right)(0.79\,\text{ft})^2 + \left(\frac{0.2\,\text{lb}}{32.2\,\text{ft/s}^2}\right)(0.25\,\text{ft})^2\right)(5\,\text{rad/s}) = \left(\frac{1}{3}\left(\frac{1.2\,\text{lb}}{32.2\,\text{ft/s}^2}\right)(0.79\,\text{ft})^2 + \left(\frac{0.2\,\text{lb}}{32.2\,\text{ft/s}^2}\right)(0.25\,\text{ft})^2\right)\omega_f$$

$$0.008141\,(5) = 0.01163\omega_f \qquad \omega_f = \boxed{3.50\,\text{rad/s}}$$

The separation of slider C happens instantaneously without any impulsive forces or moments, so the angular speed of the system is maintained.

6 Problem 18.84



The homogeneous square plate of mass m is suspended from a pin at A. The plate is at rest when it is struck by the small projectile of mass $0.2m$ traveling vertically with velocity v_0. Assuming that the projectile becomes embedded in the plate, determine the angular velocity of the plate immediately after impact.

Solution: During the collision, there are large forces between the bullet and the plate and between the plate and the pin at A. Fortunately, A doesn't move, so we can place our pivot point there. The initial angular momentum about A is $h_i = (0.2m)\,v_0\left(\frac{3}{4}\frac{b}{\sqrt{2}}\right) = 0.1061mbv_0$. In the final state, the system has a moment of inertia of

$$I_A = \frac{1}{12}m\left(2b^2\right) + m\left(\frac{b}{\sqrt{2}}\right)^2 + (0.2m)\left(\frac{3}{4}b\right)^2 = \frac{187}{240}mb^2 = 0.7792mb^2$$

So, the final angular speed is found from $0.1061mbv_0 = 0.7792mb^2\omega_f$ and $\omega_f = \boxed{0.1362\frac{v_0}{b}}$.