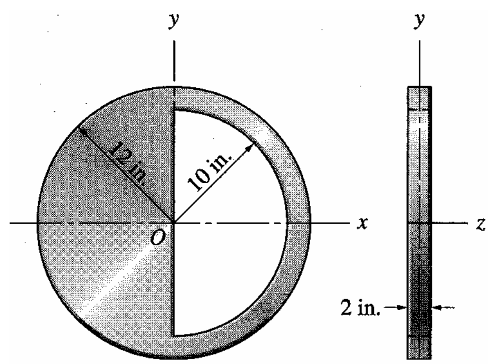
Phys202 - Dynamics - Spring 2007 HW17 Solutions

Problems 17.6, 17.10, 17.18, 17.26, 17.66, 17.104, **Bonus:** 17.98

1 Problem 17.6



The weight density of a cast aluminum wheel is 165 lb/ft^3. Find the mass center of the wheel and calculate I_z and $\overline{I}_z$.

Solution: I'll do this problem by two methods. First, the formal integration method that always works and is used then the formulas in the inside covers of the book. The wheel is best described in cylyndrical coordinates. The most straightforward combination of integration limits is a cylindrical Disc minus a half-cylindrical Hole. The Disc exists in the range $r = 0...12$ in, $\theta = 0...2\pi$, and $z = -1...1$ in. The Hole exists in the range $r = 0...10$ in for $\theta = -\pi/2...\pi/2$. Integrals over the volume will then be:

$$\int_{\text{Body}} dV = \int_{\text{Disc}} dV - \int_{\text{Hole}} dV = \left(\int_{-1}^{1} dz \int_{0}^{12\,\text{in}} r\,dr \int_{0}^{2\pi} d\theta\right) - \left(\int_{-1}^{1} dz \int_{0}^{10\,\text{in}} r\,dr \int_{-\pi/2}^{\pi/2} d\theta\right)$$

The total volume is the volume of a 12 in. cylindar minus half the volume of a 10 in. cylindar.

$$V = \left(\pi r_2^2 - \tfrac{1}{2}\pi r_1^2\right) h = \pi\left((12\,\text{in})^2 - \tfrac{1}{2}(10\,\text{in})^2\right)(2\,\text{in}) = 288\pi - 100\pi = 188\pi\,\text{in}^3$$

The total weight is: $W_T = \rho V = \left(165\,\text{lb/ft}^3\right)\left(\frac{1\,\text{ft}^3}{12^3\,\text{in}^3}\right)\left((288\pi - 100\pi)\,\text{in}^3\right) = 86.39 - 30.00\,\text{lb} = 56.40\,\text{lb}$

The center of mass is balanced in the y-direction by symmetry. In the x-direction, the integral of the outer cylindar vanishes by symmetry. The position of the CM is determined by the integral of the hole:

$$\begin{aligned}
\overline{x} &= -\frac{1}{m_T}\int_{\text{Hole}} \rho x\,dV = -\frac{\rho}{m_T}\int_{-1\,\text{in}}^{1\,\text{in}} dz \int_{0}^{10\,\text{in}} dr \int_{-\pi/2}^{\pi/2} r d\theta\,(r\cos\theta) = -\frac{1}{188\pi\,\text{in}^3}(2\,\text{in})\left[\sin\theta\right]_{-\pi/2}^{\pi/2}\left[\frac{1}{3}r^3\right]_0^{10\,\text{in}} \\
&= -\frac{2\cdot 2\cdot 10^3}{188\pi\cdot 3} = -\frac{1000}{141\pi}\,\text{in} = -2.258\,\text{in} = -0.1881\,\text{ft}
\end{aligned}$$

By the formulas in the inside of the book, take the hole to have a negative mass at position $x_{\text{Disc}} = \frac{4}{3\pi}(10\,\text{in})$. The center of mass of the Disc is zero.

$$\overline{x} = \frac{1}{m_T}\left(m_{\text{Disc}}x_{\text{Disc}} + m_{\text{Hole}}x_{\text{Hole}}\right) = \frac{1}{56.40}\left(-30.00\cdot\frac{4}{3\pi}10\,\text{in}\right) = -2.258\,\text{in} = \boxed{-0.1881\,\text{ft}}$$

The mass moment of intertia about the z-axis is (Note, these are the integral *of* r^2, not a volume times r^2):

$$\begin{aligned}
I_z &= \int_{\text{Disc}} \rho r^2 dV - \int_{\text{Hole}} \rho r^2 dV = \left(\int_{-1}^{1} dz \int_{0}^{12\,\text{in}} r\,dr \int_{0}^{2\pi} d\theta\,\rho r^2\right) - \left(\int_{-1}^{1} dz \int_{0}^{10\,\text{in}} r\,dr \int_{-\pi/2}^{\pi/2} d\theta\,\rho r^2\right) \\
&= \left(\frac{2}{12}\,\text{ft}\right)(2\pi)\left(\frac{165\,\text{lb/ft}^3}{32.3\,\text{lb/slug}}\right)\left(\frac{(1\,\text{ft})^4}{4} - \frac{1}{2}\frac{(10/12\,\text{ft})^4}{4}\right) = \boxed{1.018\,\text{slug}\cdot\text{ft}^2}
\end{aligned}$$

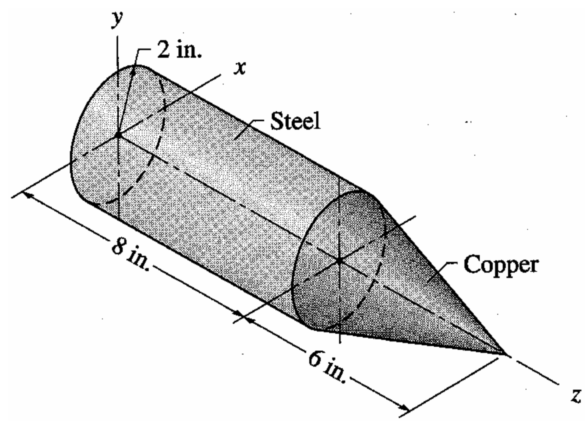
By the tables, the mass moment of inertia of a cylindar is $\frac{1}{2}mR^2$, and for a half-cylindar I_z is the same thing, because the mass is distributed in the same way.

$$I_z = \overline{I}_{z,\text{Disc}} + I_{z,\text{Hole}} = \frac{1}{2}\left(\frac{86.39}{32.2}\right)(1\,\text{ft})^2 - \frac{1}{4}\left(\frac{30.00}{32.2}\right)\left(\frac{10}{12}\,\text{ft}\right)^2 = 1.341 - 0.323 = 1.018\,\text{slug}\cdot\text{ft}^2$$

By the parallel axis theorem, the mass moment of inertia about the center of mass is

$$\overline{I}_{z,\text{Body}} = I_z - m_{\text{Body}}\overline{x}_{\text{Body}}^2 = 1.018 - \frac{56.40}{32.2}\left(0.1881^2\right) = \boxed{0.956\,\text{slug}\cdot\text{ft}^2}$$

2 Problem 17.10



The solid body consists of a steel cylinder and a copper cone. The mass density of copper is 1.10 times the mass density of steel. Locate the mass center of the body and compute $\overline{k}_x$.

Solution: The volume of each part is:

$$V_{\text{Cyl}} = \pi R^2 h = \pi\,(2\,\text{in})^2\,(8\,\text{in}) = 32\pi\,\text{in}^3 = \pi/54\,\text{ft}^3$$

$$V_{\text{Cone}} = \frac{1}{3}\pi R^2 h = \frac{1}{3}\pi\,(2\,\text{in})^2\,(6\,\text{in}) = 8\pi\,\text{in}^3 = \pi/216\,\text{ft}^3$$

Let ρ be the mass density of steel, in slug/ft^3. The mass of each part is $m_{\text{Cyl}} = \rho V_{\text{Cyl}} = (\pi/54)\,\rho \quad m_{\text{Cone}} = (1.1\rho)\,V_{\text{Cone}} = (11/10)\,(\pi/216)\,\rho \quad m_{\text{Tot}} = (17\pi/720)\,\rho$

The x and y components of the center of mass are zero. The center of mass of the cylinder is at $z_{\text{Cyl}} = 4\,\text{in}$, while the center of mass of the cone is at $z_{\text{Cone}} = 8 + h/4 = \frac{19}{2}\,\text{in} = 9.5\,\text{in}$. The overall center of mass is at:

$$\overline{z} = \left(\frac{720}{17\pi\rho}\right)\left((4\,\text{in})\,\frac{\pi\rho}{54} + \left(\frac{19}{2}\,\text{in}\right)\frac{11\pi\rho}{2160}\right) = \frac{520}{102}\,\text{in} = \frac{529}{1224}\,\text{ft} = \boxed{5.186\,\text{in} = 0.4322\,\text{ft}}$$

The mass moment of inertia about the x axis is:

$$\begin{aligned}
I_x &= I_{x,\text{Cyl}} + I_{x,\text{Cone}} = \overline{I}_{x,\text{Cyl}} + m_{\text{Cyl}}\overline{z}_{\text{Cyl}}^2 + \overline{I}_{x,\text{Cone}} + m_{\text{Cone}}\overline{z}_{\text{Cone}}^2 \\
&= \frac{1}{12}\frac{\pi\rho}{54}\left(3\,(2/12\,\text{ft})^2 + (8/12\,\text{ft})^2\right) + \frac{\pi\rho}{54}\,(4/12\,\text{ft})^2 + \frac{3}{80}\frac{11\pi\rho}{2160}\left(4\,(2/12\,\text{ft})^2 + (6/12\,\text{ft})^2\right) + \frac{11\pi\rho}{2160}\left(\frac{19}{2}/12\,\text{ft}\right)^2 \\
&= \frac{19\pi\rho}{23328} + \frac{\pi\rho}{486} + \frac{143\pi\rho}{2073600} + \frac{3971\pi\rho}{1244160} \\
&= \frac{28613\pi\rho}{4665600} = 0.006133\pi\rho = 0.01927\rho
\end{aligned}$$

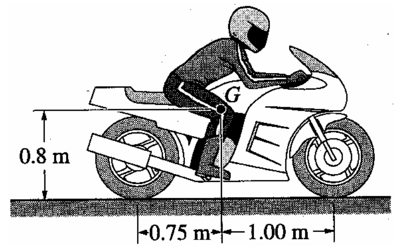
The parallel axis theorem gives the mass moment of inertia about the center of mass.

$$\begin{aligned}
\overline{I}_x &= I_x - m_{\text{Tot}}\overline{z}^2 = \frac{28613\pi\rho}{4665600} - \frac{17\pi\rho}{720}\left(\frac{529}{1224}\right)^2 \\
&= \frac{546479\pi\rho}{317260800} = 0.005411\rho
\end{aligned}$$

The radius of gyration is defined by $I_x = m_{\text{Tot}}k_x^2$, so

$$k_x = \sqrt{I_x/m_{\text{Tot}}} = \sqrt{0.005411/0.07418} = 0.270\,\text{ft} = \boxed{3.24\,\text{in}}$$

3 Problem 17.18



The combined mass center of the motorcycle and the cyclist is located at G. (a) Find the smallest acceleration for which the cyclist can perform a "wheelie" (raise the front wheel off the ground). (b) What minimum coefficient of static friction between the tires and the road is required?

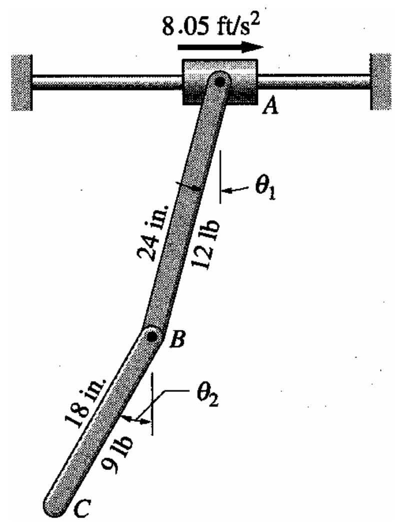
Solution: For the onset of the wheelie, the forces consist of gravity (pointing down from G), the normal force (pointing up at the rear wheel), and friction (pointing *forward* at the rear wheel). The front wheel is just barely not touching. Of these forces, only gravity causes a torque, which is CW (negative). The acceleration is in the $\hat{\imath}$ direction, and The rotational equation about the rear wheel is:

$$\begin{aligned} \sum \tau_A &= \overline{I}\alpha + \vec{r} \otimes m\vec{a} \\ -(0.75\,\mathrm{m})\, m \left(9.81\,\mathrm{m/s^2}\right) &= 0 - (0.8\,\mathrm{m})\, ma \\ a &= \boxed{9.20\,\mathrm{m/s^2}} \end{aligned}$$

Using the static $\sum F_y = 0$, the x equation becomes:

$$\begin{aligned} \sum F_x &= ma \\ \mu m \left(9.81\,\mathrm{m/s^2}\right) &= m\left(9.20\,\mathrm{m/s^2}\right) \\ \mu &= \boxed{0.9375} \end{aligned}$$

4 Problem 17.26



The two homogeneous bars are connected by a pin at B. The upper bar is pinned to the sliding collar at A. The collar has a constant acceleration of 8.05 ft/s^2 to the right. Determine the angles θ_1 and θ_2, assuming there is no oscillation (i.e. the angles are constant).

Solution: Since none of the angles are changing, that means the angular speeds of the bars are $\omega_{AB} = \omega_{BC} = 0$, and their angular accelerations are $\alpha_{AB} = \alpha_{BC} = 0$ as well. This means $\vec{a}_B = \vec{a}_A + \vec{a}_{B/A} = \vec{a}_A + \vec{\omega}_{AB} \otimes \vec{v}_{B/A} + \vec{\alpha}_{AB} \otimes \vec{r}_{B/A} = \vec{a}_A = 8.05\hat{\imath}\,\mathrm{ft/s^2}$. In the same way, $\vec{a}_C = 8.05\hat{\imath}\,\mathrm{ft/s^2}$, as does the center of mass for each of the beams.

Bar BC: If we take the torques about the center of mass (technically, since the center of mass is accelerating, we are using a point in space that coincides with the center of mass at the instant we're analyzing), we can use the easier torque formula. The gravity force has no moment arm. If we let $\vec{F}_B$ be the force on bar BC at point B, we get:

$$\begin{aligned} \sum M_G &= \overline{I}_{BC}\alpha_{BC} \\ \vec{r}_{B/BC} \otimes \vec{F}_B &= 0 \\ \vec{F}_B &\parallel \vec{r}_{B/BC} \end{aligned}$$

In other words, the pin force at B must be parallel to bar BC. The components come from Newton's Second Law for bar BC, and by dividing, we get the angle:

$$\tan\theta_2 = \frac{F_{Bx}}{F_{By}} = \frac{a_{Bx}}{g} = 0.25 \quad \rightarrow \quad \theta_2 = \boxed{14.04^\circ}$$

The actual magnitude can come from either component, for example, $F_{By} - W_{BC} = F_B\cos\theta_2 - W_{BC} = 0$, so $F_B = (9\,\mathrm{lb})\,/\cos(14.04^\circ) = 9.277\,\mathrm{lb}$. The components are $F_{Bx} = F_B \sin\theta_2 = 2.250\,\mathrm{lb}$ and $F_{By} = 9\,\mathrm{lb}$.

Bar AB: We can do the same analysis for bar AB, but now the force due to the bottom bar actually exerts a torque. Let $\vec{F}_A$ be the force on bar AB at point A. We can find the components of $\vec{F}_A$ by Newton's Second Law. The y component is easy: $F_{Ay} = 21\,\text{lb}$.

$$\begin{aligned} \sum F_x &= ma = F_{Ax} - F_{Bx} \\ \tfrac{12\,\text{lb}}{32.2\,\text{ft/s}^2}\left(8.05\,\text{ft/s}^2\right) &= F_{Ax} - (9.28\,\text{lb})\sin(14.04°) \\ F_{Ax} &= 5.250\,\text{lb} \end{aligned}$$

Now that all of the forces are known, we can find the angle θ_1 by taking the sum of the torques about the center of gravity.

$$\begin{aligned} \sum M_G &= 0 \\ 0 &= -(12\,\text{in})\cos\theta_1 F_{Bx} + (12\,\text{in})\sin\theta_1 F_{By} - (12\,\text{in})\cos\theta_1 F_{Ax} + (12\,\text{in})\sin\theta_1 F_{Ay} \\ (F_{Ax} + F_{Bx})\cos\theta_1 &= (F_{Ay} + F_{By})\sin\theta_1 \\ \tan\theta_1 &= \frac{(F_{Ax} + F_{Bx})}{(F_{Ay} + F_{By})} = \frac{5.250 + 2.250}{21 + 9} = 0.250 \\ \theta_1 &= \boxed{14.04°} \end{aligned}$$

5 Problem 17.66

[Needs Figure]

The pin B attached to the end of the uniform 8 oz crank AB slides in a vertical slot in the 12 oz slider CD. A constant counter-clockwise angular velocity of 2000 rev/min is maintained by the couple C_A. Determine C_A as a function of the crank angle θ, and use this expression to show that the gravitational forces are negligible compared with the inertial forces. Neglect friction.

Solution: Since we know the speed, we can do the kinnematics first. The angular speed is $\omega = 209.4\,\text{rad/s}$. This means that the position of B is $\vec{r}_B = (2/3\,\text{ft})\left(\cos\theta\hat{i} + \sin\theta\hat{j}\right)$, the velocity of B is $\vec{v}_B = (2/3\,\text{ft})\left(209.4\,\text{s}^{-1}\right)\left(-\sin\theta\hat{i} + \cos\theta\hat{j}\right)$, and the acceleration of B is $\vec{a}_B = -29232\left(\cos\theta\hat{i} + \sin\theta\hat{j}\right)$. With the vertical slot, the horizontal velocity and acceleration of B are the same as those of the slider.

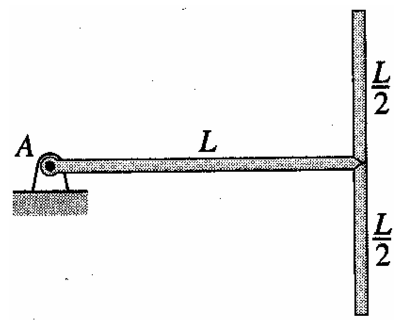
Slider CD: In the x direction, $\sum F_x = ma_x = \frac{12/16\,\text{lb}}{32.2\,\text{ft/s}^2}(-29232\cos\theta) = -680.9\cos\theta\,\text{lb}$. The only force is that of the pin at B, so this force is much larger than the 0.75 lb gravitational force.

Rod AB: Since the rod rotates about a fixed point, we can take the sum of torques about A. The angular acceleration is zero, so we don't even need to know the moment of inertia!

$$\begin{aligned} \sum M_A = 0 &= C_A - (8/16\,\text{lb})(4/12\,\text{ft})\cos\theta - (680.9\cos\theta\,\text{lb})(8/12\,\text{ft})\sin\theta \\ C_A &= (0.1667\,\text{ft}\cdot\text{lb})\cos\theta + (454.0\,\text{ft}\cdot\text{lb})\sin\theta\cos\theta \\ C_A &= \boxed{(0.167\,\text{ft}\cdot\text{lb})\cos\theta + (227.0\,\text{ft}\cdot\text{lb})\sin(2\theta)} \end{aligned}$$

Since C_A must balance the 227.0 ft·lb torque from the slider, they are both larger than the 0.167 ft·lb gravitational torque.

6 Problem 17.104



The T-bar consists of two identical rods, each of mass m and length L. Determine the pin reaction at A immediately after the bar is released from rest in the position shown.

Solution: First consider the rotation about A to find the acceleration. Then use that to find the force at A.

The mass moment of inertia is $I_A = \frac{1}{12}mL^2 + m\left(\frac{L}{2}\right)^2 + \frac{1}{12}mL^2 + mL^2 = \frac{17}{12}mL^2$. The torque equation is

$$\begin{aligned} \sum M_A &= I_A\alpha \\ -mg\frac{L}{2} - mgL &= \frac{17}{12}mL^2\alpha \\ \alpha &= -\frac{12\cdot 3\cdot g}{17\cdot 2\cdot L} = -\frac{18\cdot g}{17\cdot L} \end{aligned}$$

The center of mass is at $\overline{x} = \frac{1}{2m}\left(m\frac{L}{2} + mL\right) = \frac{3}{4}L$. The center of mass accelerates with only tangential acceleration at $a = \frac{3}{4}L\alpha = \frac{27}{34}g$ in the $-\hat{\jmath}$ direction. Then, Newton's Second Law is $\sum F_x = 0$, so there is no $\hat{\imath}$ force at A, and

$$\begin{aligned} \sum F_y &= m_T a_y \\ F_A - 2mg &= -2m\left(\frac{27}{34}g\right) \\ F_A &= \boxed{0.412mg} \end{aligned}$$