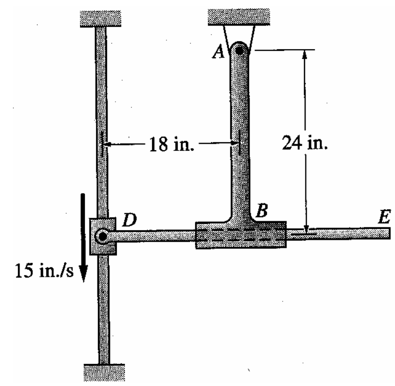
Phys202 - Dynamics - Spring 2007 HW16 Solutions

Problems 16.86, 16.109, 16.110, and 16.116. **Bonus:** Problem 16.117 for 3 points

1 Problem 16.86



Collar D of the mechanism is moving downward with the constant speed 15 in/s. For the position shown, determine the velocity and acceleration of rod DE relative to the collar B. (Hint: $\omega_{DE} = \omega_{AB}$.)

Solution: Constraints: The hint is constraint (1). The other constraints are: (2) point B, which is part of bar AB, has a fixed length and (3) Point D is moving downward with a constant velocity. With no forces, we can use whatever units we want. I'm using inches, inches/s, inches/s^2, rad/s, and rad/s^2.

Points: Point A is not moving, so $\vec{v}_A = 0$ and $\vec{a}_A = 0$.

Point D is moving down at a given speed, so $\vec{v}_D = -15\hat{\jmath}$ and $\vec{a}_D = 0$

Point B is the point of interest. It is part of bar AB and touches bar DE.

Objects: Since both bars rotate at the same rate, let $\omega \equiv \omega_{AB} = \omega_{DE}$ and $\alpha \equiv \alpha_{AB} = \alpha_{DE}$. The vectors are $\vec{\omega} = \omega\hat{k}$ and $\vec{\alpha} = \alpha\hat{k}$.

Bar AB**:** The rigidness of bar AB is best expressed by using the relative velocity equation for the rotating bar. $\vec{v}_B = \vec{v}_A + \vec{\omega} \otimes \vec{r}_{B/A}$

Since $\vec{v}_A = 0$, this leads to $\vec{v}_B$ in terms of ω_{AB}, which could have been written directly: $\vec{v}_B = 24\omega\,\hat{\imath}$

The acceleration can come from the relative acceleration equation: $\vec{a}_B = \vec{a}_A + \vec{\omega} \otimes \vec{v}_{B/A} + \vec{\alpha} \otimes \vec{r}_{B/A}$

Solve this or the acceleration from polar coordinates could be used: $\vec{a}_B = 24\omega^2\hat{\jmath} + 24\alpha\,\hat{\imath}$

Bar DE**:** Now that points B and D are known, they can be related using the relative motion equation. Point B' is the point on bar DE that coincides with point B at the given instant.

$$\vec{v}_B = \vec{v}_D + \vec{v}_{B'/D} + \vec{v}_{B/DE}$$

The second term on the right is from polar coordinates: $\vec{v}_{B'/D} = 18\omega\,\hat{\jmath}$

The last term is the relative motion, and the constraint says: $\vec{v}_{B/DE} = v_{B/DE}\hat{\imath}$

Putting it all together: $24\omega\,\hat{\imath} = (-15\hat{\jmath}) + (18\omega\hat{\jmath}) + (v_{B/DE}\hat{\imath})$

Solving the $\hat{\jmath}$ components gives: $\omega = 15/18 = 0.8333$

Solving the $\hat{\imath}$ components gives (minus) one of the desired answers: $\boxed{v_{B/DE} = 20}$

The acceleration equation for bar DE: $\vec{a}_B = \vec{a}_D + \vec{a}_{B'/D} + \vec{a}_{B/DE} + \vec{a}_{\text{Cor}}$

The acceleration of B is $\vec{a}_B = 24\alpha\hat{\imath} + 16.67\hat{\jmath}$

The acceleration of point B' relative to D is: $\vec{a}_{B'/D} = -18\omega^2\hat{\imath} + 18\alpha\hat{\jmath} = -12.5\hat{\imath} + 18\alpha\hat{\jmath}$

The rigorous way to do that would be to write a relative acceleration equation, but since we "know" how to do the two components of acceleration in polar coordinates, we can just write it down.

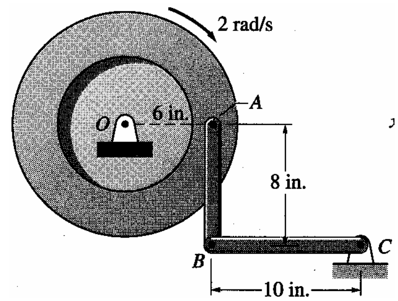
The coriolis acceleration is: $\vec{a}_{\text{Cor}} = 2\vec{\omega} \otimes \vec{v}_{B/DE} = 2\left(0.8333\hat{k}\right) \otimes \left(20\hat{\imath}\right) = 33.33\hat{\jmath}$

Putting it all together: $24\alpha\hat{\imath} + 16.67\hat{\jmath} = 0 + \left(-12.5\hat{\imath} + 18\alpha\hat{\jmath}\right) + \left(a_{B/DE}\hat{\imath}\right) + \left(33.33\hat{\jmath}\right)$

The $\hat{\jmath}$ component gives: $16.67 = 18\alpha + 33.33 \rightarrow \alpha = -0.9256$

The $\hat{\imath}$ component gives: $-22.21 = -12.5 + a_{B/DE} \rightarrow \boxed{\vec{a}_{B/DE} = -9.71\hat{\imath}}$

2 Problem 16.109



The disc is rotating clockwise with the constant angular velocity of 2 rad/s. Determine the angular accelerations of bars AB and BC in the position shown.

Solution: Each of the points is fastened to the objects, so there is no relative motion to worry about.

Point A has constant $\dot{\theta}$ about O, so there is just a rotational velocity and a centripetal acceleration $\vec{v}_A = -12\hat{j}$ and $\vec{a}_A = -24\hat{i}$

Point B is rotating about C, so $\vec{v}_B = -10\omega_{BC}\hat{j}$ and $\vec{a}_B = 10\omega_{BC}^2\hat{i} - 10\alpha_{BC}\hat{j}$

The displacement from A to B is $\vec{r}_{B/A} = -8\hat{j}$

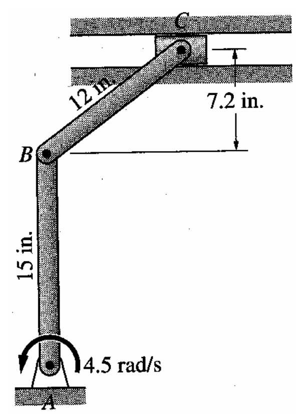
For **Bar** AB, the relative velocity equation is

$$\begin{aligned} \vec{v}_B &= \vec{v}_A + \vec{v}_{B/A} \\ -10\omega_{BC}\hat{j} &= -12\hat{j} + 8\omega_{AB}\hat{i} \\ \omega_{AB} = 0 &\quad \text{and} \quad \omega_{BC} = 1.2 \end{aligned}$$

This means that $\vec{a}_B = 14.4\hat{i} - 10\alpha_{BC}\hat{j}$. The relative acceleration equation is

$$\begin{aligned} \vec{a}_B &= \vec{a}_A + \vec{a}_{B/A} = \vec{a}_A + \vec{\omega}_{AB} \otimes \vec{v}_{B/A} + \vec{\alpha} \otimes \vec{r}_{B/A} \\ 14.4\hat{i} - 10\alpha_{BC}\hat{j} &= -24\hat{i} + 8\alpha_{AB}\hat{i} \\ \boxed{\alpha_{BC} = 0} &\quad \text{and} \quad \boxed{\alpha_{AB} = 4.8} \end{aligned}$$

3 Problem 16.110



Bar AB of the mechanism rotates with constant angular velocity of 4.5 rad/s. Determine the angular acceleration of bar BC and the acceleration of slider C at the instant shown.

Solution: Point B is rotating about fixed point A so $\vec{v}_B = -67.5\hat{i}$. The angular acceleration is zero, so $\vec{a}_B = -303.75\hat{j}$.

Point C is confined to a horizontal line, so $\vec{v}_C = v_C\hat{i}$ and $\vec{a}_C = a_C\hat{i}$.

The relative velocity of C with respect to B is
$\vec{v}_{C/B} = \vec{\omega} \otimes \vec{r}_{C/B} = \left(\omega\hat{k}\right) \otimes \left(9.6\hat{i} + 7.2\hat{j}\right) = 9.6\omega\hat{j} - 7.2\omega\hat{i}$.

The relative velocity equation is

$$\begin{aligned} \vec{v}_C &= \vec{v}_B + \vec{v}_{C/B} \\ v_C\hat{i} &= -67.5\hat{i} + 9.6\omega\hat{j} - 7.2\omega\hat{i} \end{aligned}$$

This leads to $\omega = 0$ and $v_C = -67.5$.

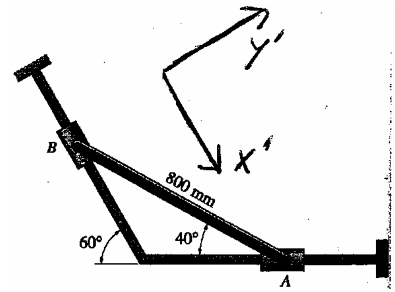
The relative acceleration of C with respect to B is purely from angular acceleration, since the angular velocity of BC is zero. $\vec{a}_{C/B} = \alpha\hat{k} \otimes \left(9.6\hat{i} - 7.2\hat{j}\right) = 9.6\alpha\hat{j} - 7.2\alpha\hat{i}$

The relative acceleration equation is

$$\begin{aligned} \vec{a}_C &= \vec{a}_B + \vec{a}_{C/B} \\ a_C\hat{i} &= -303.75\hat{j} + 9.6\alpha\hat{j} - 7.2\alpha\hat{i} \end{aligned}$$

The solution is $\boxed{\alpha = 31.64}$ and $\boxed{a_C = -227.8}$.

4 Problem 16.116



In the position shown, the angular velocity and acceleration of the rod AB are 4 rad/s and 6 rad/s^2, both counter-clockwise. For this position, calculate (a) the velocity of collar A; and (b) the acceleration of collar A.

Solution: Doing this in a rotated coordinate system in which B only moves in the x' direction eliminates B from the y' equations. The velocities are $\vec{v}_A = v_{Ax'}\hat{i}' + v_{Ay'}\hat{j}' = v_A\left(\cos 60°\hat{i}' + \sin 60°\hat{j}'\right)$ and $\vec{v}_B = v_B\hat{i}'$.

The inclination of the bar with respect to the x' axis is 20°. So, the position of A with respect to B is $\vec{r}_{A/B} = (800\,\text{mm})\left(\cos 20°\hat{i}' + \sin 20°\hat{j}'\right) = 751.75\hat{i}' + 273.62\hat{j}'$.

The relative velocity equation is

$$\begin{aligned}\vec{v}_A &= \vec{v}_B + \vec{\omega} \otimes \vec{r}_{A/B}\\ v_A \cos 60°\hat{i}' + v_A \sin 60°\hat{j}' &= v_B\hat{i}' + \left(4\hat{k}\right) \otimes \left(751.75\hat{i}' + 273.62\hat{j}'\right)\\ v_A \cos 60°\hat{i}' + v_A \sin 60°\hat{j}' &= v_B\hat{i}' + 3007.0\hat{j}' - 1094.5\hat{i}'\end{aligned}$$

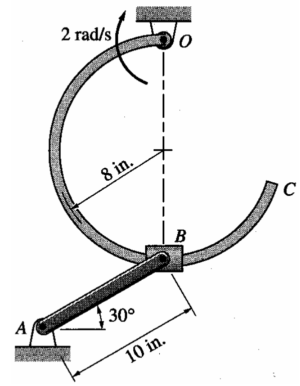
Solving the $\hat{j}'$ component yields $v_A = 3472\,\text{mm/s}$

The relative acceleration equation is

$$\begin{aligned}\vec{a}_A &= \vec{a}_B + \vec{\omega} \otimes \vec{v}_{A/B} + \vec{\alpha} \otimes \vec{r}_{A/B}\\ a_A\left(\cos 60°\hat{i}' + \sin 60°\hat{j}'\right) &= a_B\hat{i}' + \left(4\hat{k}\right) \otimes \left(3007.0\hat{j}' - 1094.5\hat{i}'\right) + \left(6\hat{k}\right) \otimes \left(751.75\hat{i}' + 273.62\hat{j}'\right)\\ a_A\left(\cos 60°\hat{i}' + \sin 60°\hat{j}'\right) &= a_B\hat{i}' - 12030\hat{i}' - 4378\hat{j}' + 4511\hat{j}' - 1642\hat{i}'\end{aligned}$$

Solving the $\hat{j}'$ equation yields $a_A = 153.6\,\text{mm/s}^2$

5 Bonus: Problem 16.117



The curved, slender bar OC rotates about O. At the instant shown, the angular velocity of OC is 2 rad/s and its angular acceleration is zero. Find the angular acceleration of bar AB at this instant.

Solution: This problem is the reason for the complicated relative motion method. You can imagine that as the bar is rotated, the point of the bar directly under O (where B is now) will get closer to O, pulling on the bar AB and increasing the 30° angle.

The relative motion equation for point B relative to bar OC is:

$$\vec{v}_B = \vec{v}_O + \vec{v}_{B'/O} + \vec{v}_{B/OC}$$

Since point B is orbiting A, $\qquad \vec{v}_B = 10\omega_{AB}\angle 120° = 10\omega_{AB}\left(\cos 120°\hat{\imath} + \sin 120°\hat{\jmath}\right)$

The anchor O isn't moving: $\qquad \vec{v}_O = 0$

Point B' is rotating with known motion: $\qquad \vec{v}_{B'/O} = -32\hat{\imath}$

And point B is restricted to move along the bar: $\qquad \vec{v}_{B/OC} = v_{B/OC}\hat{\imath}$

Putting it together:

$$-5\omega_{AB}\hat{\imath} + 8.66\omega_{AB}\hat{\jmath} = -32\hat{\imath} + v_{B/OC}\hat{\imath}$$

Solving the x equation gives $\omega_{AB} = 0$, then the y equation gives $v_{B/OC} = 32\,\text{in/s}$.

The relative acceleration equation is:

$$\vec{a}_B = \vec{a}_O + \vec{a}_{B'/O} + \vec{a}_{B/OC} + \vec{a}_{\text{Cor}}$$

With $\omega_{AB} = 0$, the acceleration of B is $\qquad \vec{a}_B = -5\alpha_{AB}\hat{\imath} + 8.66\alpha_{AB}\hat{\jmath}$

$\vec{a}_O = 0$ and point B' is orbiting: $\qquad \vec{a}_{B'/O} = 16\omega_{OC}^2\hat{\jmath} = 64\hat{\jmath}$

The constraint of B moving along the curved bar means that it has a radial and a tangential acceleration. The relative acceleration is: $\qquad \vec{a}_{B/OC} = a_{B/OC}\hat{\imath} + \left(v_{B/OC}^2/8\right)\hat{\jmath} = a_{B/OC}\hat{\imath} + 128\hat{\jmath}$

The Coriolis force is: $\qquad \vec{a}_{\text{Cor}} = 2\left(-2\hat{k}\right) \otimes \left(32\hat{\imath}\right) = -128\hat{\jmath}$

Putting it all together:

$$-5\alpha_{AB}\hat{\imath} + 8.66\alpha_{AB}\hat{\jmath} = 64\hat{\jmath} + a_{B/OC}\hat{\imath} + 128\hat{\jmath} - 128\hat{\jmath}$$

And the angular acceleration of the bar is $\boxed{\alpha_{AB} = 7.39\,\text{rad/s}^2}$.