Phys202 - Dynamics - Spring 2007 HW15 Solutions

Problems 16.6, 16.12, 16.20, 16.24, 16.26

1 Problem 16.6

The angular acceleration of a rotating disc is $\alpha = 6t\,\text{rad/s}^2$, where t is in seconds. The angular velocity of the disc is 27 rad/s clockwise when $t = 0$. Determine the total angle turned by the disc between $t = 0$ and $t = 4$ s.

Solution: Integrate the angular acceleration to find the angular velocity. $\omega = \int \alpha\, dt = 3t^2 + C$

Use the initial condition to find the constant. $\omega(0) = C = -27$

$$\omega(t) = 3t^2 - 27$$

Note that there is a zero crossing. This means the disc changes direction. If you walk to Philly and back, your displacement is zero, but you've walked 200 miles. In this case, the zero crossing is at$\omega(t) = 3t^2 - 27 = 0$ $t = 3\,\text{s}$

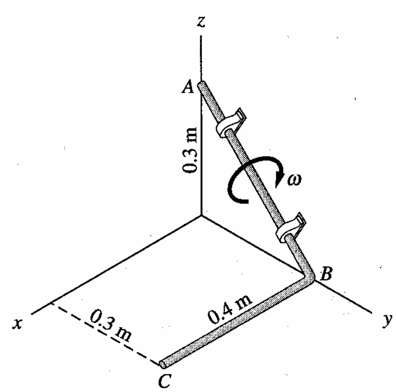
The position of the wheel is: $\theta = \int \omega\, dt = t^3 - 27t + C_2$

The total angle turned is:

$$\begin{aligned} |\theta(4) - \theta(3)| + |\theta(3) - \theta(0)| &= |-44 - -54| + |-54 - 0| \\ &= 10 + 54 = \boxed{64\,\text{rad}} \end{aligned}$$

Another way of thinking of this is that the angle turned is $\int |\omega|\, dt$.

2 Problem 16.12



The crank ABC rotates about the fixed axis AB. At the instant shown, the angular velocity of the crank is $\omega = 9\,\text{rad/s}$, and it is decreasing at the rate of 48 rad/s^2. For this instant, calculate the magnitudes of the velocity and acceleration of the end C using (a) vector equations; and (b) scalar equations.

Solution: With vector equations, we need $\vec{\omega}$. By the RHR, it points partly in the $+z$ direction and partly in the $-y$ direction. The direction is $\hat{r}_{AB} = \vec{r}_{AB}/r_{AB} = \hat{\omega} = \dfrac{-0.3\hat{j} + 0.3\hat{k}}{0.3\sqrt{2}} = \frac{\sqrt{2}}{2}\left(-\hat{j} + \hat{k}\right)$

The vector angular velocity is $\vec{\omega} = (9\,\text{rad/s})\,\frac{\sqrt{2}}{2}\left(-\hat{j} + \hat{k}\right)$

The displacement from the axis is $\vec{r}_{CB} = 0.4\hat{i}$

So, the velocity of point C is $\vec{v}_C = \vec{\omega} \otimes \vec{r}_{CB} = (0.4)(9)\,\frac{\sqrt{2}}{2}\left(\hat{k} + \hat{j}\right) = (2.546\,\text{m/s})\left(\hat{k} + \hat{j}\right)$

The speed of C is $v_C = (0.4)(9) = \boxed{3.6\,\text{m/s}}$

The normal acceleration is $\vec{a}_n = \vec{\omega} \otimes \vec{v} = (9)(3.6)\left(\frac{\sqrt{2}}{2}\right)^2\left(-\hat{i} - \hat{i}\right) = -32.4\hat{i}\,\text{m/s}^2$

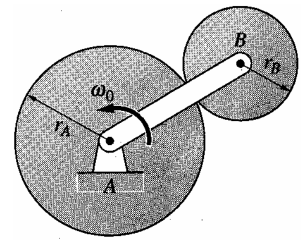
The tangential acceleration is $\vec{a}_t = \vec{\alpha} \otimes \vec{r}_{CB} = (-48)\left(\frac{\sqrt{2}}{2}\right)(0.4)\left(-\hat{j} + \hat{k}\right) \otimes \hat{i} = 13.58\left(-\hat{k} - \hat{j}\right)\,\text{m/s}^2$

The magnitude of a_t is $a_t = 48 \cdot 0.4 = 13.58\sqrt{2} = 19.2\,\text{m/s}^2$

The magnitude of the acceleration is $a = \sqrt{a_n^2 + a_t^2} = \boxed{37.66\,\text{m/s}^2}$

By the scalar equations, $v_C = R\omega = 0.4 \cdot 9 = 3.6\,\text{m/s}$. The normal acceleration is $a_n = R\omega^2 = 0.4 \cdot 9^2 = 32.4\,\text{m/s}^2$. The tangential acceleration is $a_t = R\alpha = 0.4 \cdot 49 = 19.2\,\text{m/s}^2$, and of course the magnitude of the acceleration is the same as above.

3 Problem 16.20



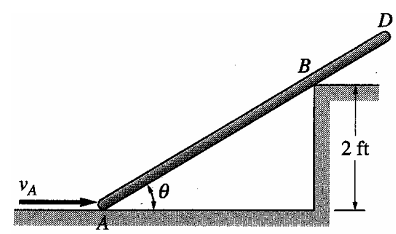
The arm joining the two friction wheels rotates with a constant angular velocity ω_0. Assuming that wheel A is stationary and that there is no slipping between the wheels, determine the angular velocity of wheel B.

Solution: Label point C as the point of A and B that are in contact. Because wheel A is not moving, $\vec{v}_C = 0$. (Yes, the point of contact moves along wheel A as wheel B rolls, but the parts of A and B that are in contact don't move. As B rolls, different points on each wheel come into contact.) We also know the motion of the center of wheel B, $v_B = (r_A + r_B)\,\omega_0$ in the clockwise direction. We can write the velocity of point C as $\vec{v}_C = \vec{v}_B + \vec{\omega} \otimes \vec{r}_{CB}$

All of the components along the bar are zero, so take the components perpendicular to the bar. The rotation term comes in negative because $\vec{r}_{CB}$ points toward the center.

$$v_C = v_B - r_B\omega_B = (r_A + r_B)\,\omega_0 - r_B\omega_B \qquad \omega_B = \boxed{\frac{r_A + r_B}{r_B}\omega_0}$$

4 Problem 16.24



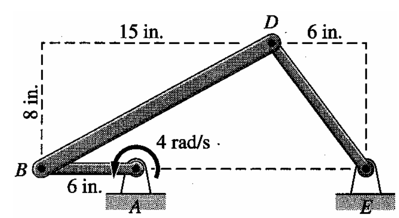
End A of bar AD is pushed to the right with the constant velocity $v_A = 1.2\,\text{ft/s}$. (a) Determine the angular velocity of AD as a function of θ. (b) By differentiating the result of part (a), find the angular acceleration of AD.

Solution: Do the problem in a tilted x'-y' coordinate system, with x' along AD. The x' component of $\vec{v}_A$ is $v_{Ax'} = 1.2\cos\theta$, and $v_{Ay'} = -1.2\sin\theta$. The relative velocity $\vec{v}_{BA} = \vec{\omega} \otimes \vec{r}_{BA} = \omega\frac{2}{\sin\theta}\left(\hat{k} \otimes \hat{\imath}'\right) = \frac{2\omega}{\sin\theta}\hat{\jmath}'$. Point B has no y' velocity, so the relative velocity equation is

$$v_{By'} = v_{Ay'} + r_{BA}\omega \qquad \rightarrow \qquad 0 = -1.2\sin\theta + \frac{2}{\sin\theta}\omega \qquad \rightarrow \qquad \omega = \boxed{(0.6\,\text{rad/s})\sin^2\theta}$$

The angular accel. is $\alpha = \dot{\omega} = 0.6\left(2\sin\theta\cos\theta\dot{\theta}\right) = 1.2\sin\theta\cos\theta\left(0.6\sin^2\theta\right) = \boxed{\left(0.72\,\text{rad/s}^2\right)\sin^2\theta\cos\theta}$

5 Problem 16.26



The link AB of the mechanism rotates with the constant angular speed of 4 rad/s counterclockwise. Calculate the angular velocities of links BD and DE in the position shown.

Solution: Use a global xy coordinate system. The velocity of point B is $\vec{v}_B = -r_{BA}\omega_{BA}\hat{\jmath} = -24\hat{\jmath}\,\text{in/s}$

The velocity of point D is $\vec{v}_D = \vec{\omega}_{DE} \otimes \vec{r}_{DE} = \omega_{DE}\hat{k} \otimes \left(-6\hat{\imath} + 8\hat{\jmath}\right) = \omega_{DE}\left(-6\hat{\jmath} - 8\hat{\imath}\right)$

The relative velocity equation for bar BD is

$$\begin{aligned} \vec{v}_D &= \vec{v}_B + \vec{\omega}_{DB} \otimes \vec{r}_{DB} \\ \vec{v}_D &= \vec{v}_B + \omega_{DB}\hat{k} \otimes \left(15\hat{\imath} + 8\hat{\jmath}\right) \\ -8\omega_{DE}\hat{\imath} - 6\omega_{DE}\hat{\jmath} &= -24\hat{\jmath} + \omega_{DB}\left(15\hat{\jmath} - 8\hat{\imath}\right) \end{aligned}$$

The x component is

$$\begin{aligned} -8\omega_{DE} &= -8\omega_{DB} \\ \omega_{DE} &= \omega_{DB} \end{aligned}$$

The y component is

$$\begin{aligned} -6\left(\omega_{DB}\right) &= -24 + 15\omega_{DB} \\ \omega_{DE} = \omega_{DB} &= 24/21 = \boxed{1.143\,\text{rad/s}} \end{aligned}$$