Phys202 - Dynamics - Spring 2007 HW14 Solutions

Problem SP-2

A wheel of radius R is attached to a car that is driving down the road at speed v. Find the velocity $\vec{v}_D$ of a clump of dirt stuck to the outside of the wheel as a function of time. Graph the components v_{Dx} and v_{Dy}, and graph the speed v_D of the dirt.

Solution: The speed of the dirt is the speed of the center of the wheel plus the speed of the dirt relative to the center. If $t = 0$ is the point when the dirt is at the top of the rotation,$\vec{v}_D = v\hat{i} + R\omega\left(\cos(\omega t)\,\hat{i} - \sin(\omega t)\,\hat{j}\right)$

The angular speed $\omega = v/R$, so the dirt's velocity is $$\vec{v}_D = v\left((1+\cos(\omega t))\,\hat{i} - \sin(\omega t)\,\hat{j}\right)$$

The magnitude is

$$\begin{aligned} v_D^2 &= (1+\cos\omega t)^2 + \sin^2\omega t \\ &= 1 + 2\cos\omega t + \cos^2\omega t + \sin^2\omega t \\ &= 2(1+\cos\omega t) \\ v_D &= \sqrt{2(1+\cos\omega t)} \end{aligned}$$