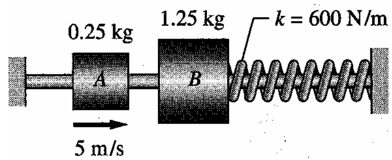
Phys202 - Dynamics - Spring 2007 HW13 Solutions

Chapter 15, Problems 106, 108, 110, 137, 149. Chapter 16, Problems 1 and 9

1 Problem 15.106



After the sliding collar A hits a stationary collar B with the speed of 5 m/s, it rebounds with a speed of 2.5 m/s, directed to the left. Determine the coefficient of restitution for the impact.

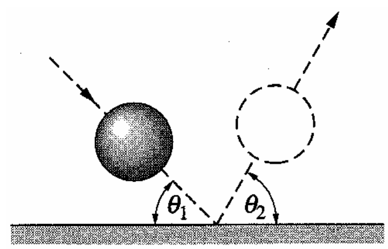
Solution: This problem deals only with the collision, and the spring does not take part. Conservation of momentum gives v_B after the collision.

$$\begin{aligned} p_i &= p_f \\ (0.25\,\mathrm{kg})\,(5\,\mathrm{m/s}) &= (0.25\,\mathrm{kg})\,(-2.5\,\mathrm{m/s}) + (1.25\,\mathrm{kg})\,v_B \\ v_B &= 1.5\,\mathrm{m/s} \end{aligned}$$

The coefficient of restitution is defined as:

$$e = \frac{v_{Bf} - v_{Af}}{v_{Ai} - v_{Bi}} = \frac{1.25 - -2.5}{5} = \boxed{0.75}$$

2 Problem 15.108



The elastic ball is bounced off a rigid surface. Show that the relationship between the angle of incidence and the rebound angle is $\tan\theta_2 = e\tan\theta_1$, where e is the coefficient of restitution. Neglect friction.

Solution: Conservation of momentum in the x direction:

$$\begin{aligned} p_{1x} &= p_{2x} \\ v_1\cos\theta_1 &= v_2\cos\theta_2 \end{aligned}$$

Definition of e:

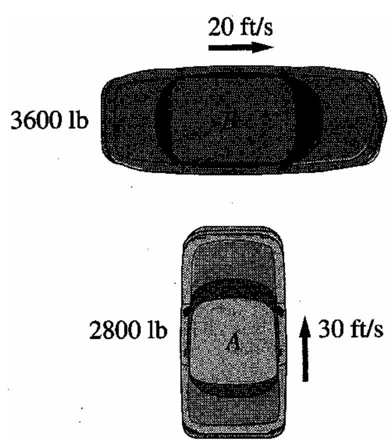
$$e = \frac{v_{2y}}{-v_{1y}}$$

$$v_1\sin\theta_1 = e v_2\sin\theta_2$$

Divide these equations to get the result.

$$\boxed{\tan\theta_2 = e\tan\theta_1}$$

3 Problem 15.110



Two cars traveling with velocities shown collide at an intersection. The coefficient of restitution is 0.25 for the impact, and the contacting forces are frictionless. Calculate the velocity of each car after the impact.

Solution: With no friction, each car independently obeys conservation of momentum in the x direction.

$$\boxed{v_{Ax} = 0}$$

$$\boxed{v_{Bx} = 20\,\mathrm{ft/s}}$$

In the y direction, there is overall conservation of momentum. Multiply both sides by g to use weights instead of masses:

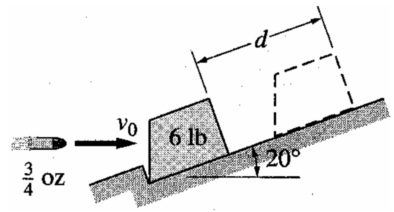
$$\begin{aligned} p_{yi} &= p_{yf} \\ (2800\,\mathrm{lb})\,(30\,\mathrm{ft/s}) &= (2800\,\mathrm{lb})\,v_{Ay} + (3600\,\mathrm{lb})\,v_{By} \end{aligned}$$

And the definition of e gives:

$$\begin{aligned} e &= \frac{v_{By} - v_{Ay}}{30\,\mathrm{ft/s}} \\ (30\,\mathrm{ft/s})\,0.25 &= v_{By} - v_{Ay} \end{aligned}$$

The solution is $\boxed{v_{Ay} = 8.91\,\mathrm{m/s}}$ $\boxed{v_{By} = 16.41\,\mathrm{m/s}}$

4 Problem 15.137



A 3/4 oz bullet strikes a stationary 6 lb block with a horizontal velocity v_0 and mecomes embedded. If the maximum displacement of the block after impact is $d = 1.2\,\text{ft}$, determine v_0. Neglect friction and assume that the block does not leave the inclined surface.

Solution: When the bullet first hits, the system will have a momentum in the x direction only. Call this state 1. Then the constraint force (normal force) causes the block/bullet combination to slide up the ramp instead of horizontally. This is basically a collision with the ramp that results in state 2, the beginning of the slide. Then, state 3 is the end of the slide.

From $0 \rightarrow 1$, conservation of momentum in the x direction:

$$\begin{aligned} p_{0x} &= p_{1x} \\ (0.75/16)\, v_0 &= (0.75/16 + 6)\, v_1 \\ v_0 &= 129 v_1 \end{aligned}$$

From $1 \rightarrow 2$, conservation of momentum in the x' direction (up the ramp) applies.

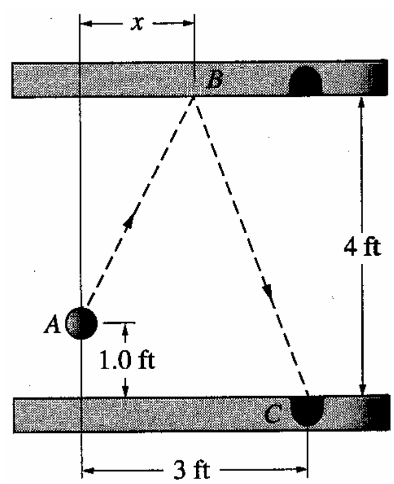
$$\begin{aligned} p_{1x'} &= p_{2x'} \\ m_T v_{1x'} = m_T v_1 \cos 20^\circ &= m_T v_2 \\ v_1 &= 1.064 v_2 \end{aligned}$$

From $2 \rightarrow 3$, conservation of energy applies.

$$\begin{aligned} E_2 &= E_3 \\ \tfrac{1}{2} m_T v_2^2 &= m_T g h_3 \\ v_2 &= \sqrt{2gd \sin 20^\circ} = 5.141\,\text{ft/s} \end{aligned}$$

In the end, $v_0 = 129 v_1 = 129\,(1.064 v_2) = 129 \cdot 1.064 \cdot 5.141\,\text{ft/s} = \boxed{706\,\text{ft/s}}$. This process with the separate state 1 could be skipped by doing the conservation of momentum of the collision in the x' direction.

5 Problem 15.149



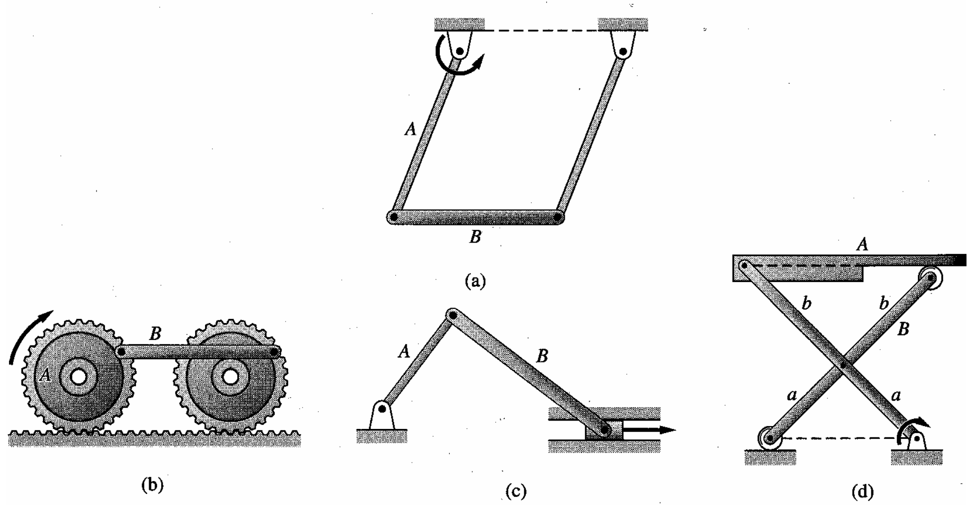
The billiard ball A is to be banked off the rail at B so that it will enter the pocket C. The coefficient of restitution for the impact between the ball and the rail is 0.85. Neglecting friction, determine x.

Solution: The impact at B is the same as problem 15.108 above. Knowing this,

$$\begin{aligned} \tan \theta_2 &= e \tan \theta_1 \\ \frac{4}{3 - x} &= 0.85 \frac{3}{x} \\ 4 \cdot x &= 0.85 \cdot 3 \cdot (3 - x) \\ x &= \boxed{1.168\,\text{ft}} \end{aligned}$$

6 Problem 16.1

16.1 Characterize the motion of bodies *A* and *B* of each mechanism shown as translation; (2) rotation about a fixed axis; or (3) general plane motion.



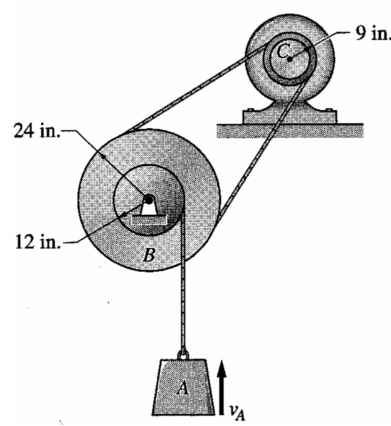
Answers: (a) A is rotational motion, B is translational.

(b) A is general motion, B is translational.

(c) A is rotational, B is general motion.

(d) A is translational, B is general motion.

7 Problem 16.9



The belt-driven hoist lifts the load A at the constant speed $v_A = 3\,\text{ft/s}$. Find the angular speeds of the pulley B and the electric motor C.

Solution: The rotation of the pulley and the movement of A are linked by $(12\,\text{in})\,\omega_B = 3\,\text{ft/s}$, so $\omega_B = 3\,\text{rad/s}$.

This causes the belt to move at $v_{\text{belt}} = (24\,\text{in})\,(3\,\text{rad/s}) = 72\,\text{in/s}$.

The motor has to spin at $(9\,\text{in})\,\omega_C = 72\,\text{in/s}$ or $\omega_C = 8\,\text{rad/s}$.