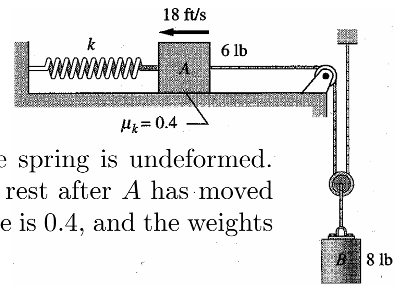
Phys202 - Dynamics - Spring 2007 HW12 Solutions

Problems 15.56, 15.60, 15.72, 15.80, 15.82.

1 Problem 15.56



In the position shown, block A is moving to the left at 18 ft/s, and the spring is undeformed. Determine the spring stiffness k that would cause the system to come to rest after A has moved 3 ft. The coefficient of kinetic friction between A and the horizontal surface is 0.4, and the weights of the pulleys are negligible.

Solution: With a non-conservative force (friction), conservation of energy doesn't quite apply, but what I call semi-conservation of energy ($\Delta E = \text{Work}_{\text{NC}}$)does work. The friction force is found from the normal force. $\sum F_{Ay} = N - W_a$ $N = 6\,\text{lb}$ $f = \mu N = 2.4\,\text{lb}$

The amount of work done by the friction force is $U_f = \vec{f} \cdot \Delta\vec{r}_A = (2.4\,\text{lb})\,\hat{\imath} \cdot (-3\,\text{ft})\,\hat{\imath} = -7.2\,\text{lb}\cdot\text{ft}$

The rope constraint is: $L = (x_{\text{pulley}} - x_A) + (y_{\text{pulley}} - y_B) + (y_{\text{ceiling}} - y_B)$

It gives the relationship between the motions of A and B: $0 = -v_A - 2v_B$

$$v_B = -\tfrac{1}{2}v_A = -0.5\,(-18\,\text{ft/s}) = 9\,\text{ft/s}$$

The $\Delta\vec{r}$'s have the same relationship as the velocities. $\Delta y_B = -0.5\Delta x_A = -0.5\,(-3\,\text{ft}) = 1.5\,\text{ft}$

The initial kinetic energies are: $T_A = \tfrac{1}{2}m_A v_A^2 = \tfrac{1}{2}\left(\frac{6\,\text{lb}}{32.2\,\text{ft/s}^2}\right)(18\,\text{ft/s})^2 = 30.18\,\text{lb}\cdot\text{ft}$

$$T_B = \tfrac{1}{2}m_B v_B^2 = \tfrac{1}{2}\left(\frac{8\,\text{lb}}{32.2\,\text{ft/s}^2}\right)(9\,\text{ft/s})^2 = 10.06\,\text{lb}\cdot\text{ft}$$

The potential energy changes are: $\Delta V_{\text{spring}} = \tfrac{1}{2}k\delta_f^2 - \tfrac{1}{2}k\delta_i^2 = \tfrac{1}{2}k\,(3\,\text{ft})^2 = 4.5k$

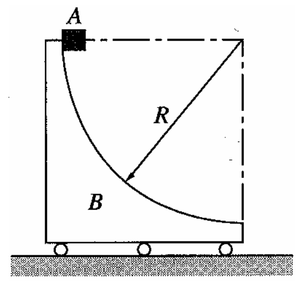
$$\Delta V_{B,\text{grav}} = m_B g \Delta y_B = (8\,\text{lb})\,(1.5\,\text{ft}) = 12\,\text{lb}\cdot\text{ft}$$

Knowing the work done by non-conservative forces, kinetic and potential energy are now useful. The initial kinetic energy is known. The initial potential energy is zero, because the spring is unstretched and by taking the initial height of B to be zero. The final kinetic energy is zero, and the final potential energy can be determined.

$$\begin{aligned} E_f - E_i &= U_{\text{NC}} \\ \Delta V_{\text{spring}} + \Delta V_{B,\text{grav}} + \Delta T_A + \Delta T_B &= U_f \\ 4.5k + 12 - (30.18 + 10.06) &= -7.2 \end{aligned}$$

$$\boxed{k = 4.676\,\text{lb/ft}}$$

2 Problem 15.60



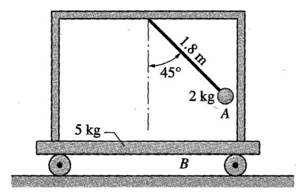
Particle A of mass m_A is released from rest in the position shown and slides with negligible friction down the quarter-circular track of radius R. The body B of mass m_B containing the track can slide freely on the horizontal surface. Determine the speeds of A and B when A reaches the bottom of the track.

Solution: Conservation of energy tells us how much total kinetic energy the system must have when A reaches the bottom, but there will be two unknowns (v_A and v_B). Conservation of momentum in the x-direction will give the additional necessary equation.

$$\begin{aligned} E_i &= E_f \\ m_A g R &= \tfrac{1}{2} m_A v_A^2 + \tfrac{1}{2} m_B v_B^2 \\ m_A g R &= \tfrac{1}{2} m_A v_A^2 + \tfrac{1}{2} m_B \left(v_A \tfrac{m_A}{m_B} \right)^2 \\ m_A g R &= \tfrac{1}{2} m_A \left(1 + \tfrac{m_A}{m_B} \right) v_A^2 \\ v_A &= \sqrt{\frac{2gR}{\left(1 + \frac{m_A}{m_B}\right)}} = \boxed{\sqrt{\frac{2gRm_B}{(m_B + m_A)}}} \end{aligned}$$

$$\begin{aligned} p_{xi} &= p_{xf} \\ 0 &= m_A v_A + m_B v_B \\ v_B &= -v_A \tfrac{m_A}{m_B} \\ v_B &= \boxed{-\sqrt{\frac{2gRm_A^2}{m_B (m_B + m_A)}}} \end{aligned}$$

3 Problem 15.72



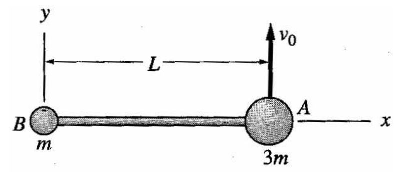
The system is at rest when the simple pendulum is released from the position shown. When the pendulum reaches the vertical position for the first time, compute the absolute speed of (a) the pendulum's bob A; and (b) the carriage B. Neglect rolling resistance of the carriage.

Solution: This is the **exact same as the previous problem**, except that the change in height isn't R, it is $R(1 - \cos\theta)$. (Two objects, one falls, constraint is relative circular motion, no net work by constraint, no external x forces.) The velocities are

$$v_A = \sqrt{\frac{2(9.81\,\mathrm{m/s^2})\big((1.8\,\mathrm{m})\big(1-\sqrt{2}/2\big)\big)(5\,\mathrm{kg})}{((2\,\mathrm{kg})+(5\,\mathrm{kg}))}} = \boxed{2.718\,\mathrm{m/s}}$$

$$v_B = -\tfrac{2\,\mathrm{kg}}{5\,\mathrm{kg}}\,(2.718\,\mathrm{m/s}) = \boxed{-1.087\,\mathrm{m/s}}$$

4 Problem 15.80



The two particles A and B, connected by a rigid rod of negligible mass, are initially at rest on a smooth horizontal surface. If A is given the initial velocity v_0 as shown, determine the velocities of A and B when the assembly has rotated through 90°.

Solution: We know the initial conditions (mass A moving with $\vec{v}_{Ai} = v_0\hat{j}$ and mass B not moving). We want the final conditions, $\vec{v}_A$ and $\vec{v}_B$, which is 4 variables. Our 4 equations will be conservation of momentum (2 equations, one for the x-component and one for the y-component), conservation of angular momentum (1 equation because there's only planar motion which means only a z-component of $\vec{h}$), and the constraint ($r_{AB} = \text{const}$).

First, conservation of momentum: $$\vec{p} = (3m)\left(v_0\hat{j}\right) = m_A\vec{v}_A + m_B\vec{v}_B$$

$$\boxed{0 = 3mv_{Ax} + mv_{Bx}} \text{ and } \boxed{3mv_0 = 3mv_{Ay} + mv_{By}}$$

The constraint means: $$r_{AB} = |\vec{r}_A - \vec{r}_B| = \text{const} = \sqrt{(r_{Ax} - r_{Bx})^2 + (r_{Ay} - r_{By})^2}$$

$$0 = \frac{1}{2r_{AB}}\left[2\left(r_{Ax} - r_{Bx}\right)\left(v_{Ax} - v_{Bx}\right) + 2\left(r_{Ay} - r_{By}\right)\left(v_{Ay} - v_{By}\right)\right] = \tfrac{1}{r_{AB}}\left(\vec{v}_{AB} \cdot \vec{r}_{AB}\right)$$

$$\vec{v}_{AB} \cdot \vec{r}_{AB} = 0$$

Which means the components of velocity in the direction of $\vec{r}_{AB}$ are equal $$\vec{v}_A \cdot \vec{r}_{AB} = \vec{v}_B \cdot \vec{r}_{AB}$$

$$\boxed{v_{Ay} = v_{By}}$$

The angular momentum about the center of mass: $$\vec{h}_{CM} = \left(\frac{L}{4}\right)(3m)\,v_0$$

In the final state, the ang. mom. is $$\tfrac{3}{4}mLv_0 = \left(\tfrac{L}{4}\right)(3m)\left(-v_{Ax}\right) + \left(\tfrac{3L}{4}\right)(m)\left(v_{Bx}\right)$$

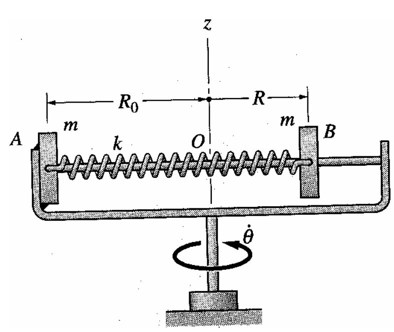
$$\boxed{v_o = v_{Bx} - v_{Ax}}$$

Sovle these 4 equations for v_{Ax}, v_{Ay}, v_{Bx}, and v_{By}.

$$\boxed{v_{Ay} = v_{By} = \frac{3}{4}v_0} \qquad v_0 = \left(-3v_{Ax}\right) - v_{Ax} \qquad \boxed{v_{Ax} = \frac{-1}{4}v_0} \qquad \boxed{v_{Bx} = \frac{3}{4}v_0}$$

In vector form: $$\boxed{\vec{v}_A = \frac{1}{4}v_0\left(-\hat{\imath} + 3\hat{\jmath}\right)} \qquad \boxed{\vec{v}_B = \frac{1}{4}v_0\left(3\hat{\imath} + 3\hat{\jmath}\right)}$$

5 Problem 15.82



The assembly consisting of two identical masses and a supporting frame of negligible mass rotatesfreely about the z-axis. Mass A is attached to the frame, but mass B is free to slide on the horizontal bar. An ideal spring of stiffness k and free length R_0 is connected between A and B. Determine all possible combinations of R and $\dot{\theta}$ for which B remains at rest relative to the frame.

Solution: Use polar coordinates and consider only mass B. The force of the spring is always pointing toward the center (i.e. $-\hat{e}_R$-direction) because its free length is the radius of the arm. In the R direction, $\sum F_R = -kR = ma_R = -mR\dot{\theta}^2$. This equation is true when $\boxed{R = 0}$ or when $\dot{\theta}^2 = k/m$ which means $\boxed{\dot{\theta} = \sqrt{\frac{k}{m}}}$.