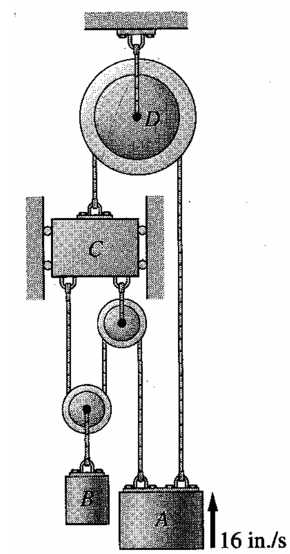
Phys202 - Dynamics - Spring 2007 HW11 Solutions

Problems 15.17, 15.23, 15.28, 15.35, 15.36, 15.38, 15.66.

1 Problem 15.17



Determine the velocity of block B at the instant when the velocity of block A is 16 in/s, directed upward.

Solution: There are two ropes, so there are two constraint equations. I'll define all positions from a common fixed origin (which I don't have to specify because everything is differences in heights), with positive upward. Also, I'm not going to worry about the difference in height between B and the pulley it's attached to (or between C and the pulley hanging from it) because those don't change with time. L_1 is the upper-right rope, and L_2 is the lower-left rope.

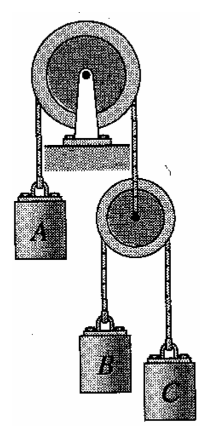
$$L_1 = (y_D - y_C) + (y_D - y_A) \qquad L_2 = (y_C - y_B) + (y_C - y_B) + (y_C - y_A)$$

The next step is to take the time derivative. Note $\dot{L}_1 = \dot{L}_2 = \dot{y}_D = 0$. This gives 2 equations for 3 unknowns.

$$0 = -v_C - v_A \qquad 0 = 3v_C - 2v_B - v_A$$

The final equation is $v_A = 16\,\text{in/s}$. From the left equation, $v_C = -v_A = -16\,\text{in/s}$. Then, $0 = 3(-16) - 2v_B - 16$, so $v_B = \boxed{-32\,\text{in/s}}$.

2 Problem 15.23



At a certain instant, the velocity of block A is 0.2 m/s and the velocity of block B relative to block C is 0.6 m/s, both directed downward. Determine the velocities of B and C at this instant.

Solution: Label everything as positive upward relative to a global coordinate system, call the pulley D, and skip straight to the derivatives. Our givens are $v_A = -0.2\,\text{m/s}$, $v_B - v_C = -0.6\,\text{m/s}$. The two equations from the ropes are:

$$0 = (0 - v_A) + (0 - v_D) \qquad 0 = (v_D - v_B) + (v_D - v_C)$$

The left equation gives $v_D = -v_A = 0.2\,\text{m/s}$. This turns the right equation into $v_B + v_C = 2v_D = 0.4\,\text{m/s}$. The solution is $\boxed{v_C = 0.5\,\text{m/s}}$ and $\boxed{v_B = -0.1\,\text{m/s}}$.

3 Problem 15.28

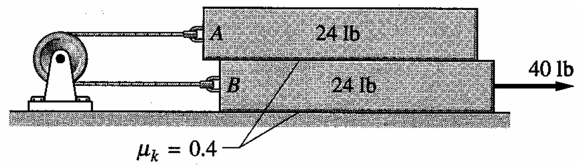
A 0.08 lb bullet is fired from a rifle that is clamped to a trolley. The combined weight of the rifle and trolley is 20 lb. After firing, the velocity of the trolley is known to be $v_A = 4.8\,\text{ft/s}$ to the left. Calculate (a) v_B, the velocity of the bullet after firing; and (b) the muzzle velocity (the velocity of the bullet relative to the barrel of the rifle).

Solution: With no external forces during the firing, conservation of momentum applies. The initial momentum is $p_i = 0$. The final momentum is $p_f = m_A v_A + m_B v_B = 0$, so

$$v_B = -\frac{m_A}{m_B} v_A = -\frac{20}{0.08}(-4.8\,\text{ft/s}) = \boxed{1200\,\text{ft/s}}$$

The muzzle velocity is $v_B - v_A = 1200 - (-4.8) = \boxed{1205\,\text{ft/s}}$.

4 Problem 15.35



Determine the tension in the cable connecting blocks A and B after the constant 40 lb force is applied.

Solution: First analyze block A. As B is pulled to the right, A will slide to the left. Therefore, the friction force counters this motion and pushes A to the right. The normal force holding A counters only the weight, so they're equal and the friction force is $f_A = \mu_k W = 0.4\,(24\,\text{lb}) = 9.6\,\text{lb}$. Newton's Second Law in the horizontal direction for the top block is

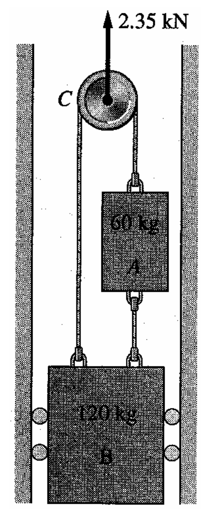
$$\sum F_x = m_A a_A = f_A - T = 9.6\,\text{lb} - T = \frac{24\,\text{lb}}{32.2\,\text{ft/s}^2} a_A$$

For block B, the normal force counters the weight of B and the reaction to the normal force on A, so $f_B = 0.4\,(48\,\text{lb}) = 19.2\,\text{lb}$ to the left. Note that there is also the reaction to the friction force on A, f_A pointing to the left. Newton's Second Law in the horizontal direction for block B is

$$\sum F_x = m_B a_B = 40\,\text{lb} - T - f_B - f_A = 40\,\text{lb} - T - 19.2\,\text{lb} - 9.6\,\text{lb} = 11.2\,\text{lb} - T = \frac{24\,\text{lb}}{32.2\,\text{ft/s}} a_B$$

These are two equations for three unknowns, and the remaining equation comes from the constraint that $-a_A = a_B \equiv a$. Then add the equations to get $\boxed{T = 10.4\,\text{lb}}$.

5 Problem 15.36



The system of blocks A and B, and the weightless pulley C, is pulled upward by the constant 2.35 kN force. Determine the force in the cable joining A and B.

Solution: The tension in the upper rope comes from $\sum F_{\text{pulley}} = 2.35\,\text{kN} - 2T_1$, so $T_1 = 1175\,\text{N}$.

The two blocks move together (bottom rope), so there is one common acceleration a.

For block A:

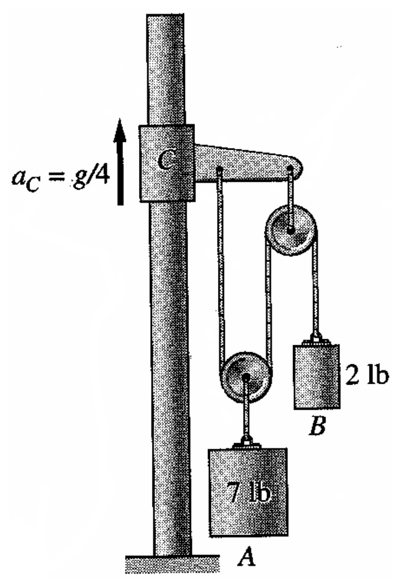
$$\sum F_A = (1175\,\text{N}) - T_2 - (60\,\text{kg})\,g = (60\,\text{kg})\,a$$

For block B:

$$\sum F_B = (1175\,\text{N}) + T_2 - (120\,\text{kg})\,g = (120\,\text{kg})\,a$$

The solution is $T_2 = 1175\,\text{N}/3 = \boxed{391.7\,\text{N}}$.

6 Problem 15.38



The acceleration of the sliding collar C is $g/4$, directed upward. Determine the accelerations of blocks A and B. Neglect the masses of the pulleys.

Solution: Again, by expressing all heights from a common origin, write the length of the rope as the sum of the segments, and take the time derivative to get

$$0 = (v_C - v_A) + (v_C - v_A) + (v_C - v_B) = 3v_C - 2v_A - v_B,$$

therefore

$$0 = 3a_C - 2a_A - a_B$$

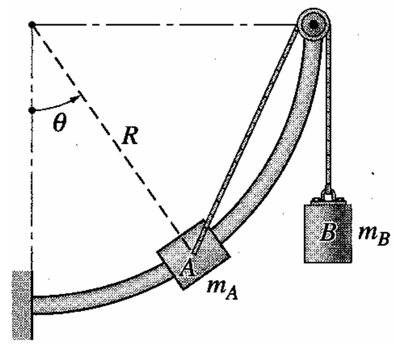
For block A, $\sum F_A = 2T - (7\,\text{lb}) = \frac{7\,\text{lb}}{32.2\,\text{ft/s}^2} a_A$

For block B, $\sum F_B = T - (2\,\text{lb}) = \frac{2\,\text{lb}}{32.2\,\text{ft/s}^2} a_B$

The last equation is the given info: $a_C = \left(32.2\,\text{ft/s}^2\right)/4$

The solution is: $a_A = 0$ $\quad a_B = 24.15\,\text{ft/s}^2$

7 Problem 15.66



The system is released from rest when $\theta = 0$. Determine the ratio m_A/m_B of the two masses for which the system will come to rest again when $\theta = 60°$. Neglect friction.

Solution: Since this problem relates velocity (zero initial and final speed) and position, work-energy should be used. There are no non-conservative forces, so conservation of energy for the system as a whole applies:

$$m_A g y_{Ai} + m_B g y_{Bi} = m_A g y_{Af} + m_B g y_{Bf}$$
$$m_A g \Delta y_A + m_B g \Delta y_B = 0.$$

The key here is to relate everything to the angle θ. The height of A is easy. $y_A = -R\cos\theta$ in both the initial and final states.

To get the change in height of block B, the length of rope is used as a contraint. The length of the segment stretching from A to the pulley is easily calculated with the law of cosines, $C^2 = A^2 + B^2 - 2AB\cos(\phi) = R^2 + R^2 - 2R^2\sin(\theta)$, or $C = \sqrt{2R^2(1-\sin\theta)}$. The length of the rope is $L = C + (y_{\text{pulley}} - y_B)$, and it is constant. Thus, $y_B = C(\theta) + y_{\text{pulley}} - L$. $\Delta y_B = \Delta C = R\sqrt{2}\left(\sqrt{1-\sin\theta_f} - \sqrt{1-\sin\theta_i}\right)$. The conservation of energy formula then becomes:

$$\begin{aligned}
-m_A g \Delta y_A &= m_B g \Delta y_B \\
m_A g\left(R\cos\theta_f - R\cos\theta_i\right) &= m_B g R\sqrt{2}\left(\sqrt{1-\sin\theta_f} - \sqrt{1-\sin\theta_i}\right) \\
m_A\left(\frac{1}{2} - 1\right) &= m_B\sqrt{2}\left(\sqrt{1-\sqrt{3}/2} - 1\right) = -0.8966 m_B \\
m_A/m_B &= \boxed{1.793}
\end{aligned}$$