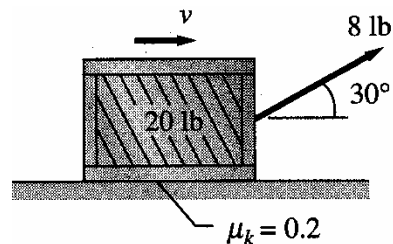


Problems 14.114, 14.115, 14.117, 14.118, 14.119, 14.122, 14.123, and 14.124.

1 Problem 14.114

A constant 8 lb force is pulling on a 20 lb crate across a horizontal surface [at an angle of 30° above the horizontal]. If the coefficient of kinetic friction between the crate and the surface is 0.2, determine the time required to increase the speed v of the crate from 10 ft/s to 40 ft/s.



Solution: We know all of the forces. Let \vec{F} be the 8 lb force; let \vec{W} be the weight; let N be the normal force; and let f be the friction force. We use $\vec{F} = m\vec{a}$ in rectangular coordinates to find Δt . In rectangular coordinates, $a_x = \dot{v}_x$ and $a_y = \dot{v}_y$. In the y direction, there is no motion.

$$\sum F_y = 0 = N - W + F \sin 30^\circ \qquad N = W - F \sin 30^\circ$$

Here, $W = 20$ lb, and $F = 8$ lb, so N is “known.”

In the x direction,

$$\sum F_x = ma_x = F \cos 30^\circ - f = F \cos 30^\circ - \mu N \qquad a_x = \frac{F}{m} \cos 30^\circ - \frac{\mu N}{m}$$

The mass is $m = W/g = W/(32.2 \text{ ft/s}^2)$. So now the acceleration a_x (which I’ll call a now) is “known” in terms of N and knowns. It is a constant.

Integration of the acceleration gives the amount of time.

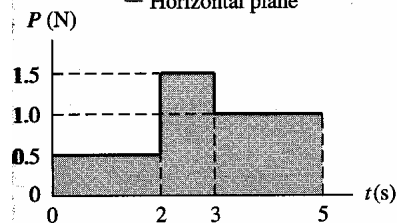
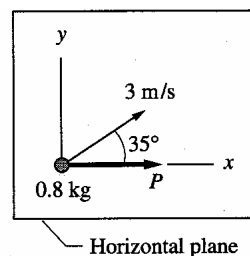
$$a = \dot{v} \qquad \int a \, dt = \int \dot{v} \, dt = \Delta v = v_f - v_i = a\Delta t \qquad \Delta t = (v_f - v_i) / a$$

It would have been just as good to use the Impulse-Momentum relationship.

Numbers: The normal force is $N = 20 - 8 \cos 30^\circ = 16$ lb. The friction force is $f = \mu_k N = 0.2(16) = 3.2$ lb. The acceleration is $a_x = \frac{1}{20/32.2} (8 \cos 30^\circ - 3.2) = 6.002 \text{ ft/s}^2$. The time required is $\Delta t = (40 - 10) / 6.002 = 4.998$ s.

2 Problem 14.115

A 0.8 kg particle slides across a frictionless horizontal plane. A force P applied to the particle acts in the x -direction with a magnitude that varies with time as shown in the P - t diagram. If the velocity of the particle at time $t = 0$ is 3 m/s in the direction shown, determine the speed when $t = 5$ s.



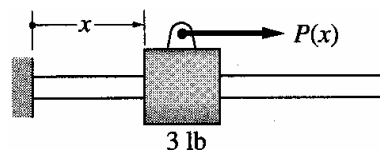
Solution: Since this is a known 1-D time-varying force, the Impulse-Momentum relationship in rectangular coordinates works. The initial velocity is $\vec{v}_i = (3 \text{ m/s}) (\cos 35^\circ \hat{i} + \sin 35^\circ \hat{j})$ and the momentum is $\vec{p} = m\vec{v}$. The impulse caused by the force is

$$\vec{L}_{i \rightarrow f} = \int \vec{F} \, dt = \hat{i} (1 + 1.5 + 2) = 4.5 \hat{i} \text{ N} \cdot \text{s} \qquad \vec{p}_f = \vec{p}_i + \vec{L}_{i \rightarrow f} = m\vec{v}_f$$

Numbers: The final momentum is $\vec{p}_f = 0.8(3) (\cos 35^\circ \hat{i} + \sin 35^\circ \hat{j}) + 4.5 \hat{i} \rightarrow p_f = 6.611 \text{ kg} \cdot \text{m/s}$. The final speed is $v_f = 6.611 / 0.8 = 8.264 \text{ m/s}$.

3 Problem 14.117

A 3 lb collar is pulled along a frictionless rod by a force $P(x) = 0.3/x^2$ lb, where x is in feet. When $x = 1.2$ ft, the velocity of the collar is 2 ft/s to the right. When $x = 4$ ft, determine (a) the velocity of the collar; and (b) the power of the force P .



Solution: Since the force varies with position and we want the velocity at a given position, calculate the work and relate it to the kinetic energy.

$$U_{i \rightarrow f} = \int \vec{F} \cdot d\vec{r} = \int_{1.2}^4 0.3x^{-2} dx = \Delta KE = \frac{1}{2}m(v_f^2 - v_i^2)$$

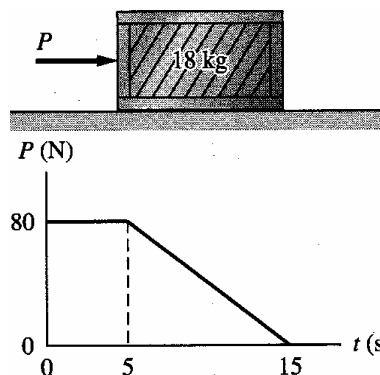
The final velocity v_f is the requested quantity. It is important to make sure that the final kinetic energy is positive. The work is positive and the initial kinetic energy is positive, which means that the final kinetic energy is indeed positive. The power of a force is

$$P = \vec{F} \cdot \vec{v} = \frac{0.3}{x^2} \cdot v_f$$

Numbers: The work is $U_{i \rightarrow f} = 0.175$ lb · ft. The initial kinetic energy is $T_i = \frac{1}{2}mv_i^2 = \frac{1}{2} \left(\frac{3 \text{ lb}}{32.2 \text{ ft/s}^2} \right) (2 \text{ ft/s})^2 = 0.1863$ lb · ft. The final kinetic energy is $T_f = 0.3613$ lb · ft. The final speed is $v_f = \sqrt{2(0.3613)/(3/32.2)} = 2.785$ ft/s. The power at the final location is $P = \frac{0.3}{4^2} (2.785) = 0.0522$ lb · ft/s.

4 Problem 14.118

The coefficients of static and kinetic friction between a 18 kg crate and a horizontal surface are $\mu_s = 0.3$ and $\mu_k = 0.25$, respectively. The crate is at rest when a horizontal, time-dependent force P is applied. Determine the speed of the crate at $t = 10$ s.



Solution: From the graph, the integral of the force is $\int_0^{10} P dt = 5 \cdot 80 + 5 \cdot (80 + 40)/2 = 700$ N · s. The normal force is equal to the weight, so the maximum static friction force is $f_{\max} = \mu_s W = 0.3(18 \text{ kg})(9.81 \text{ m/s}^2) = 52.97$ N, therefore the box slides. The kinetic friction force is $f = \mu_k W = 0.25(18 \text{ kg})(9.81 \text{ m/s}^2) = 44.145$ N. The total force on the crate is $\sum F = P - f$, and the total impulse is $L = \int P dt - \int f dt = (700 \text{ N} \cdot \text{s}) - (44.145 \text{ N})(10 \text{ s}) = 258.6 \text{ N} \cdot \text{s}$. This impulse causes a change in momentum $\Delta p = m\Delta v$, so the final velocity is $v_f = 258.6/18 = \boxed{14.36 \text{ m/s}}$.

5 Problem 14.119

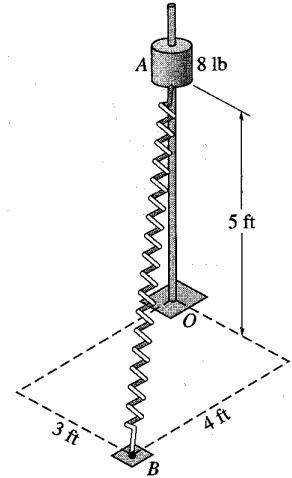
An 8 lb collar slides on a vertical rod with negligible friction. A spring has a free length $L_0 = 3$ ft, a stiffness of $k = 4.5$ lb/ft, and is connected from the collar to the point $(3\text{ ft}, 4\text{ ft}, 0)$. If the mass is released from rest from $(0, 0, 5\text{ ft})$, determine its speed at $(0, 0, 0)$.

Solution: Conservation of energy applies.

$$E_i = E_f$$

$$\frac{1}{2}k(L_i - L_0)^2 + mgh_i = \frac{1}{2}k(L_f - L_0)^2 + \frac{1}{2}mv_f^2 + mgh_f$$

It's important to make sure the final kinetic energy is positive. The initial spring energy is $V_i = \frac{1}{2}k(L_i - L_0)^2 = \frac{1}{2}(4.5\text{ lb/ft})(\sqrt{2} \cdot 5\text{ ft} - 3\text{ ft})^2 = 37.29\text{ lb} \cdot \text{ft}$. The final spring energy is $V_f = \frac{1}{2}k(L_f - L_0)^2 = \frac{1}{2}(4.5\text{ lb/ft})(5\text{ ft} - 3\text{ ft})^2 = 9\text{ lb} \cdot \text{ft}$. The gravitational potential energy changes by $\Delta V_g = (8\text{ lb})(-5\text{ ft}) = -40\text{ lb} \cdot \text{ft}$. The decrease in potential energy is the final kinetic energy $T_f = 68.29\text{ lb} \cdot \text{ft}$. The final speed is $v_f = \sqrt{2(68.29\text{ lb} \cdot \text{ft}) / (8\text{ lb}/32.2\text{ ft/s}^2)} = \boxed{23.45\text{ ft/s}^2}$



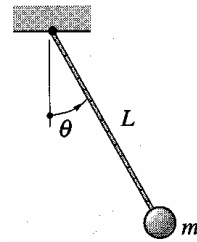
6 Problem 14.122

Use the moment-angular momentum relationship to show that the differential equation describing the motion of a pendulum is $\ddot{\theta} + (g/L)\sin\theta = 0$.

Solution: There are two forces on the pendulum. The tension is in the (negative) radial direction, while gravity is both in the radial and θ directions. Newton's Second Law for the θ direction is

$$\sum F_\theta = -mg \sin\theta = ma_\theta = m(R\ddot{\theta} + 2\dot{R}\dot{\theta}) = mR\ddot{\theta}$$

Noting that $R = L$, the equation of motion is $\ddot{\theta} = -(g/L)\sin\theta$.



7 Problem 14.123

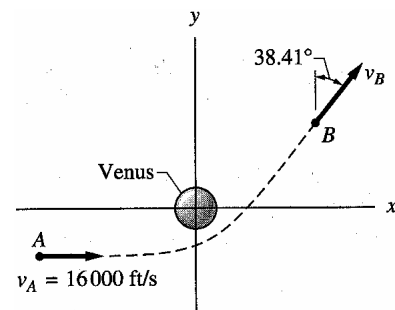
An unpowered space vehicle of mass m travels past the planet Venus from point A at $(-5.0, -1.0) \times 10^8$ ft to point B at $(3.960, 3.382) \times 10^8$ ft. If the speed at A is 16000 ft/s, determine the speed at B.

Solution: Conservation of energy applies. The mass of Venus is $4.869 \times 10^{24}\text{ kg} = 3.336 \times 10^{21}\text{ slug}$. The initial potential energy is $V_i = -G\frac{mM}{R} = -m(3.439 \times 10^{-8})(4.869 \times 10^{24}) / (\sqrt{26.0} \times 10^8) = -m(2.250 \times 10^5)$. The final potential energy is $V_f = -G\frac{mM}{R} = -m(3.439 \times 10^{-8})(4.869 \times 10^{24}) / (\sqrt{3.96^2 + 3.382^2} \times 10^8) = -m(2.203 \times 10^5)$.

The initial kinetic energy is $T_i = \frac{1}{2}mv_i^2 = m(1.28 \times 10^8)$, so the final kinetic energy is $T_f = T_i + V_i - V_f = m(1.28 \times 10^8 - 2.250 \times 10^5 + 2.203 \times 10^5) = m1.279953 \times 10^8 = \frac{1}{2}mv_f^2$, so $v_f = 15999.7\text{ ft/s}$.

Alternative Solution: Use conservation of angular momentum. $\vec{h}_i = \vec{r} \otimes m\vec{v} = (-5\hat{i} - 1\hat{j}) \times 10^8 \otimes m(16000\hat{j}) = m \cdot 1.6 \times 10^{12}\hat{k}\text{ ft}^2/\text{s}$. The final angular momentum is $\vec{h}_f = (3.960\hat{i} + 3.382\hat{j}) \times 10^8 \otimes mv_f(\sin 38.41^\circ\hat{i} + \cos 38.41^\circ\hat{j}) = (3.103 \times 10^8 - 2.101 \times 10^8)mv_f\hat{k} = \vec{h}_i$, so $v_i = 15970\text{ m/s}$.

The discrepancy may be due to roundoff error in the quoted position and angle.



8 Problem 14.124

A 6 kg mass hits the free end of a spring with a speed v_0 , and the maximum deformation of the spring of stiffness $k = 15 \text{ kN/m}$ is observed to be 0.032 m. Determine v_0 .

Solution: This is a straightforward conservation of energy problem. $\delta_i = 0$ and $\delta_f = 0.032 \text{ m}$, while $v_f = 0$.

$$E_i = E_f \quad \frac{1}{2}mv_0^2 = \frac{1}{2}k\delta_f^2 \quad v_0 = \sqrt{\frac{k}{m}}x_{\max}$$

Remember this result; it'll come back again.

Numbers: The speed at impact is $v_0 = \sqrt{(15 \times 10^3)/6}(0.032) =$
1.6 m.

