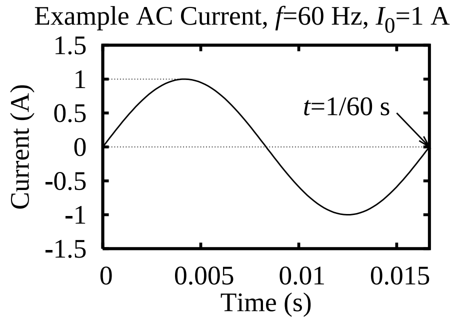
Phys132 - Fund. Physics 2 - Spring 2007 Basic AC Circuits

1 Introduction

AC electricity is used frequently because it is more versatile than DC. Sound information can be transmitted through an electric wire by using AC. AC voltages are easily stepped up or down by transformers, making AC ideal for moving energy instantantly from a big generator to our homes. Radio frequencies are used to transmit video, audio, and even digital information from one place to another. A complete analysis of AC circuits is way beyond the scope of this course, but we can cover some of the basics.

2 AC Current as a wave

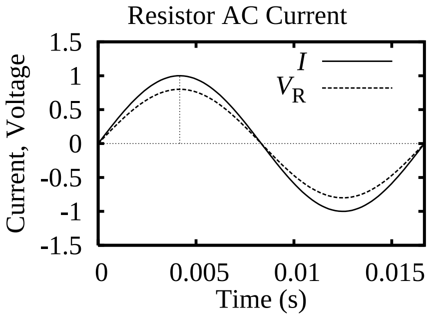


If we measured the actual current in an AC wire with an ammeter, slowed time down so it wasn't changing so fast, and graphed the current, it would look like a sine wave. The maximum current is called the amplitude or **peak current** I_0, measured in amps (A). The actual current varies between $\pm I_0$. Each complete back-and-forth of the current is called a **cycle**, and the number of cycles per second is called the **frequency** f, measured in hertz (Hz) which are cycles per second. We could also measure the time of each cycle, called the **period** $T = 1/f$. These ideas of amplitude, period, and frequency are the same as they were with waves, but now there is no position x in the equation.

$$I(t) = I_0 \sin(2\pi f t)$$

One thing that we didn't talk about much in the waves chapter is the fact that a wave can be shifted simply by starting the first cycle at a different time. Notice that with our current, the wave is both zero and increasing at time $t = 0$. We'll use this point and the actual peak (at $t = \frac{1}{4}\frac{1}{60}$ s) as reference points.

3 Resistors in an AC circuit



Ohm's Law still works for resistors. $I = V_R/R$

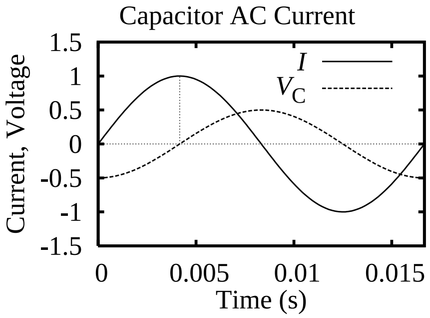
$$I_0 \sin(2\pi f t) = (V_0 \sin(2\pi f t))/R \qquad I_0 = V_0/R$$

If the current is fixed (i.e. set to a certain value), a bigger resistor will have a bigger voltage. If the voltage is fixed, a bigger resistor will have a smaller current. We say that the resistance opposes the current. Ohm's Law means that the voltage as a wave is just a scaled version of the current. Resistors don't care about frequency, so the resistor voltage is the same for a given current regardless of the frequency of the current.

- When the current I peaks, V_R peaks.
- The voltage is the same regardless of the frequency.

4 Capacitors in an AC circuit

Remember that a capacitor is a device that stores electric charge. Whenever a capacitor has a charge, there is a voltage, and vice-versa. The formula is $Q = CV$. If we try to put a DC current through a capacitor, the current I serves to charge to capacitor. Eventually the capacitor gets full and its voltage V stops the current. Another way of saying this is the capacitor blocks DC current because of the insulator between the capacitor plates.



If we put an AC current through the capacitor, then the current keeps charges and discharges the capacitor in alternating directions. Since the current keeps switching directions, the capacitor never gets a chance to actually block the current. The higher the frequency, the easier it is to get the current to flow, and a smaller voltage is needed for the capacitor. We use an Ohm's Law-like formula for AC **amplitudes** which looks like

$$V_0 = ZI_0$$

where Z is called the **impedance**. It works kind of like resistance does for DC circuits. In fact, a resistor R has an impedance $Z_\mathrm{R} = R$. Since at higher frequencies, a lower voltage across a capacitor is required to drive the same current, the impedance of a capacitor must be lower at high frequencies. The impedance of a capacitor is inversely proportional to frequency.

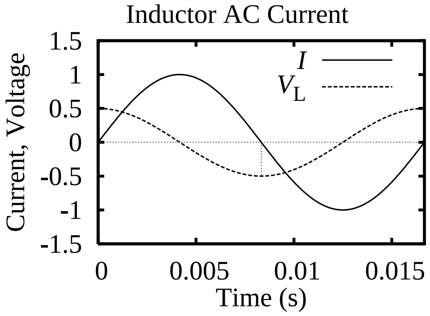
$$Z_\mathrm{C} = \frac{1}{2\pi f C}$$

But now, the voltage isn't exactly proportional to the current. With the capacitor, the current serves to charge the capacitor, which raises the voltage. When the current gets to zero (as it crosses the axis), the capacitor voltage flattens out, and as the current is negative, the voltage decreases. The capacitor voltage is no longer a sine wave, but a negative cosine wave with the same frequency.

$$V_C = -Z_C I_0 \cos(2\pi f t)$$

- Peak current makes the voltage increase (vertical dashed line). This makes the current shift to the left (delayed voltage).
- Higher frequencies mean lower capacitor impedance and higher capacitor current.

5 Inductors in an AC circuit



Inductors work by creating a magnetic field with their current. The magnetic field through the coil of the inductor forms a flux. Then, any change in the current changes the magnetic field, which changes the flux. A changing flux induces a voltage. **Inductors oppose change** in the current, so as the current is **increasing**, the inductor has a **large positive voltage**. The decreasing current causes the biggest negative voltage (marked with a vertical line). At high frequencies, the current is changing the most, so the inductor requires the most voltage and has the highest impedance. Inductor impedance is proportional to frequency.

$$Z_L = 2\pi f L$$

- Increasing current makes the voltage peak positive, which makes the current shift to the right (delayed).
- High frequencies mean higher inductor impedance and lower inductor voltage.

6 Series RLC circuit and Resonance

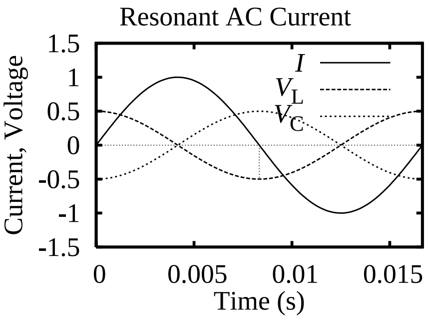
When a resistor R, an inductor L, and a capacitor C, are placed together so the current *must* flow through each (no branches), they form a series RLC circuit. The current that flows through one component must go through the next; it cannot stop. The actual energy of any given charge (voltage is energy per charge) flowing around the circuit increases in the power source, then changes with each component, eventually getting back to zero where it started. So, just like with pure resistor circuits, in AC circuits:

- In a series circuit, the current I is the same in each component.
- In a series circuit, the instantaneous voltages add. This does *not* mean the amplitudes add.

The total voltage is

$$V_{\text{Tot}} = V_R + V_L + V_C = RI_0 \sin(2\pi f t) + Z_L I_0 \cos(2\pi f t) - Z_C I_0 \cos(2\pi f t)$$

It's not trivial to add these sines and cosines together, but the rules of trigonometry do provide a way. We won't go into the gory details now. But do notice that there are two terms with $I_0 \cos(2\pi f t)$ in them. These terms can be combined to get $(Z_L - Z_C)\, I_0 \cos(2\pi f t)$. The inductor and capacitor have impedances that can cancel each other out. This is called **resonance**. This happens only at one particular frequency



$$\begin{aligned} Z_L &= Z_C \\ 2\pi f L &= \frac{1}{2\pi f C} \\ 2\pi f &= \frac{1}{\sqrt{LC}} \end{aligned}$$

At this resonant frequency, the inductor and capactor "disappear" and stop impeding the flow of current. Then, the most current will flow through the circuit. This effect is counter-intuitive, because the inductor is impeding the current a little and the capacitor is impeding the current a little. But, since their voltages are in opposite directions (look at the figures above, the voltages are minus each other), the net effect is zero.

The shift in current of an RLC circuit depends on which impedance is the strongest. At low frequencies, the capacitor's impedance is high, so the shift of current is to the left (the voltage is delayed), while at high frequencies, the inductor's impedance is high and the shift of current is to the right. At the resonant frequency, there is no net shift.

7 Lab 209 Explanation

In lab 209, you were given boxes with an R_1R_2, RL, RC, or RLC circuit in them. A fixed voltage was placed across the circuit, which caused a certain current. You measured the voltage of one resistor (the green terminal) with Channel 2 of your oscilloscope. This measurement also represents the current through the circuit, because the resistor voltage is proportional to the current. So, you can think of Channel 2 as the Current. You also measured the total voltage with Channel 1. Observations you should have made are:

- With the R_1R_2 circuit, the current didn't depend on frequency, and there was never any shift in time.
- With the RL circuit, the current was smallest at high frequencies. The current (Ch. 2) was shifted to the right as compared to the voltage (Ch. 1).
- With the RC circuit, the current was smallest at low frequencies. There was a shift in current (Ch. 2) to the left as compared to the voltage (Ch. 1).
- With the RLC circuit, the current was small at very low and very high frequencies. The shift was to the left at low frequencies (where the capacitor was dominant) and to the right at high frequencies (where the inductor was dominant). At the **resonant frequency**, the current (Ch. 2) was maximum and there was no shift.