Homework # 6

Due in class Wednesday, March 19.

Unless otherwise noted, all problems on this homework are to be done without reference to calculating devices of any kind.

[**1**] Correct and resubmit any incorrect problems from In-Class Quiz #1. Successfully doing this will earn you half of the homework points.

[**2**] If you would like a rapid review of series solutions to differential equations, skim pp. 565-569 in *Arfken & Weber*.

[**3**] We will now hunt for series solutions of the following differential equation:

$$\frac{d^2 f}{dx^2} - 2x\frac{df}{dx} + 2pf = 0\,.$$

In this expression, p is a positive integer or zero.

Attempt a series solution of the form $f(x) = \sum_{n=0}^{\infty} a_n x^{k+n}$, where k is some undetermined constant. By considering the lowest powers of x that appear in the differential equation, arrive at a set of restrictions involving k, a_0, and a_1.

[**4**] By forcing the coefficient of each x^n power to vanish independently, arrive at a recursion relation between different a_n coefficients.

[**5**] Use your results from Parts [**3**] and [**4**] so show that non-zero k values *do not* add additional solutions beyond those obtained with $k = 0$.

[**6**] Suppose now that $p = 6$. Show that your recursion relation leads to one finite power series solution for $f(x)$ and one infinite power series solution for $f(x)$. Write out your finite series solution explicitly. After you have obtained your finite series solution, plug it back into your differential equation and show that it works!

[**7**] Provide an expression here for your infinite power series solution in the case $p = 6$. You will obviously have to express this as an infinite sum.

[**8**] Suppose now that $p = 7$. Show that your recursion relation leads to one finite power series solution for $f(x)$ and one infinite power series solution for $f(x)$. Write out your finite series solution explicitly.

[**9**] Provide an expression here for your infinite power series solution in the case $p = 7$. You will obviously have to express this as an infinite sum.

[**10**] We are happy with our finite series solutions (which obviously converge), but have some concern about the infinite ones. Convergence can be guaranteed if the coefficients in our infinite series get small *fast enough.* To determine whether our series converge or not, calculate the limit

$$\mathcal{R} \equiv \lim_{n\to\infty} \left(\frac{|\mathrm{Term}_{n+2}|}{|\mathrm{Term}_n|} \right)$$

for the infinite series that you found in Parts [**7**] and [**9**]. In this expression, Term_n refers to the entire nth term in each series, *including* the xs. You may assume that x is finite when you take your limit.

[**11**] The test for convergence is simple. If $\mathcal{R}$ is a well-defined limit less than 1, your series converges. So...do your infinite series converge for finite values of x?