Chapter 5 notes

Review of chapters 1-4

In general, the whole point of chapters 1-4 was to introduce three motion variables (position, velocity and acceleration) and develop mathematical relationships between them and time based upon the definitions:

$$\begin{aligned}\vec{v} &= d\vec{s}/dt \\ \vec{a} &= d\vec{v}/dt\end{aligned}$$

From these two simple definitions, we can obtain specific relationships. For example, we know that the average velocity and average acceleration can be expressed as follows:

$$\begin{aligned}\vec{v}_{\mathrm{avg}} &= \Delta\vec{s}/\Delta t \\ \vec{a}_{\mathrm{avg}} &= \Delta\vec{v}/\Delta t\end{aligned}$$

and we can show that if the acceleration is constant, the following relationship holds:

$$\Delta\vec{s} = \vec{v}_{\mathrm{i}}\Delta t +^1/_2\vec{a}(\Delta t)^2$$

Mathematics is not the entire story, though. We have to have strong understanding of what position, velocity and acceleration are so that we can properly approach a problem. Otherwise, we are stuck using the "plug and chug" method (looking for an equation that gives us the variable being asked for) and that is a very inefficient and, ultimately ineffective, method.

A crucial item to recognize about postiion, velocity and acceleration is that they are *vectors*, which means that they have a direction. If all three are parallel then we have *one-dimensional* motion. In that case, we can use plus and minus to indicate direction. And from the two definitions, we can construct graphs of position, velocity and acceleration vs. time and use the graphs to relate the variables.

However, when all three are in different directions, we have *two-dimensional* (or more) motion. In that case, we must write the equation for a particular direction. In the $\hat{x}$ direction, for example, the following expression holds for constant acceleration:

$$\Delta s_x = v_{x,\mathrm{i}}\Delta t +^1/_2 a_x(\Delta t)^2$$

If we choose two directions, $\hat{x}$ and $\hat{y}$ that are perpendicular, then we can rewrite the vector expressions twice: once for each direction. We can then treat the problem as two one-dimensional problems, using plus and minus for direction in each case.

Note: For circular motion, treat the motion as one-dimensional (tangential or angular).

Introduction to Forces

We now go beyond simply *describing* motion and will explain *why* the motion in certain situations is as it is. To do this, we need to introduce forces.

For example, we have assumed that an object moving through the air experiences an acceleration of 9.8 $\mathrm{m/s^2}$ directed downward. We have not explained why this would be the case. In addition, we have not explained why this is true only if air resistance is ignored.

To explain this, we need to recognize a couple of things. First, in order to for an object to accelerate (speed up, slow down or change directions), there must be a force exerted upon it. If the acceleration is constant, that means the force exerted on it must be constant. For an object moving through the air, the force acting on it is constant if we can ignore the air resistance. In that case, the only force acting on it is what we call the gravitational force, which is independent of the object's motion and thus remains constant as it moves through the air. It is for this reason that the acceleration of the object is constant.

It also turns out that the direction of the acceleration is always in the direction of the force. The acceleration of an object, in this case, is downward because the gravitational force is downward.

Note: Chapter 5 discusses the relationship between force and acceleration. Chapter 13 (first couple of sections) discusses the gravitational force.

While this explains why the acceleration is constant, it doesn't explain why each object experiences the same value of acceleration (9.8 $\mathrm{m/s^2}$). To explain this, we need to recognize a second idea: the more massive the object, the greater the force that is necessary to accelerate the object. It turns out that the gravitational force is larger on a more massive object. So, the more massive object, being harder to accelerate, experiences a larger force. The two effects cancel, leading to the same acceleration for both heavy and light objects (assuming air resistance can be ignored).

Note: Chapter 6 discusses how to predict an object's acceleration in different situations.

It turns out that the relationship between force and acceleration is rather simple. To make predictions about an object's acceleration, however, requries that we be able to identify the forces acting on the object. This requires us to have a clear idea of what forces are and how to identify them. That is a big part of what chapters 5 and 6 are about.

Note: You will not only need to identify what forces are acting on an object in various situations but you will also need to determine the sum of all of the forces acting on the object (since it is the sum that is ultimately related to the object's acceleration). Force is a vector, so adding forces is like adding velocities or displacements.

Newton's Laws

The relationship between forces and motion is contained in Newton's second law of motion, which can be written as follows:

$$\vec{F}_{\text{net}} = m\vec{a}$$

where the "net" subscript means that we need to add up all of the forces. The "m" refers to the mass of the object. It is crucial to recognize that this means the velocity remains constant if all of the forces acting on an object add to zero. In other words, the object *can* be moving – it just won't *change* its motion. That idea is embedded in Newton's *first* law of motion.

Newton's *third* law of motion says that forces are due to interactions between objects. So, you need two objects in order for there to be a force. And, if there is a force, each object in the interaction pair experiences the *same* force due to that interaction. In other words, if object A exerts a 10-Newton force on object B, then object B must also exert a 10-Newton force on object A.

Note: By Newton's second law, the acceleration of object A may be very different from the acceleration of object B if the two masses are very different. Newton's third law only says the forces are the same, not the accelerations.

Note: Newton's laws involve vectors. With two-dimensional problems, you must write the expressions as two one-dimensional expressions.