Chapter 9

Describing AC Circuits

Puzzle #9: What is the difference between AC and DC?

9.1 AC vs. DC

In part II, a particular potential difference was applied to a circuit. It was unchanging. Consequently, a particular, steady current was set up. Such a current is called "DC" (for direct current). Various techniques were introduced that allowed us to determine the current that would flow given a particular potential difference.

If we were restricted to use only the DC currents that we have been exploring thus far, electricity would be far less useful than it is. We could still light light bulbs, run some DC motors, and run some crude relay type computers, but many of our most useful electronic devices would not function. These devices rely on AC voltages and knowledge of AC voltages and currents is needed to understand how they work (or predict their effect on organisms).

9.2 Describing AC Voltage and Current

Typically, an AC voltage is one that changes sinusoidally in time (see volume I for information on sinusoidal motion). By this, we mean that the voltage value changes with time[1]. In particular, it undergoes a periodic cycle that is repeated over and over again.[2]

SINCE THE VOLTAGE IS OSCILLATING, WILL THE CURRENT OSCILLATE ALSO?

Yes.

In this section, I will provide the terminology we use to describe the oscillation in voltage and current (the same terminology is used for both).

9.2.1 Time

To represent how quickly the voltage oscillates, we'll use three terms: *period* (T), *frequency* (f) and *angular frequency* (ω). All three can be used and you should become comfortable switching between them.

Period (T)

The *period*, T, is the time it takes to undergo one cycle. Technically, the period has units of *time* but, for our purposes, we'll treat it as though it has units of *time per cycle*. For example, if it takes a pendulum 2 seconds to complete one cycle, we'll say the period is 2 s/cycle, even though technically the period is just 2 seconds.

Frequency (f)

The *frequency*, f, of the signal is the number of cycles that are completed in a given amount of time. For example, a signal might repeat itself 10,000 times every second. In that case, the frequency is 10,000 cycles per second.

[1]Technically, the voltage changes with time in a particular way. As shown in Volume I, the sinusoidal motion can be expressed mathematically as $A\cos(\omega t)$ or $R\cos(2\pi t/T)$.

[2]It is the cyclical nature that allows us to construct radio receivers and transmitters, to create computers that can operate at 800,000,000 operations per second, and to construct most of the modern electronic gadgets that we have become accustomed to. The digital devices we use actually utilize square wave pulses more than sinusoidal variations, but many of the features that we study here are important in both cases. The sinusoidal variation is easier to understand, so that is the one that we shall principally concern ourselves with.

To express frequency, we tend to use the unit of "Hertz" which means a cycle per second. Consequently, 10,000 cycles per second is equivalent to 10,000 Hertz.

Notice that the units of frequency are just the inverse of those for period: cycles per time, instead of time per cycle. In fact, there is a very simple relationship between frequency and period – they are inverses of one another:

$$f = \frac{1}{T} \qquad \text{and} \qquad T = \frac{1}{f}$$

The smaller the frequency, the slower the voltage oscillates up and down and the larger the period.

Angular frequency (ω)

As in volume I, we can describe cycles in lots of different units, including revolutions, degrees and radians. When the frequency is expressed in radians per time (instead of cycles per time), we call it the *angular frequency* and use a Greek letter for its variable abbreviation (the lower-case letter omega, ω).

Since one cycle is equivalent to 2π radians, the relationship between the frequency and the angular frequency is as follows:

$$\omega = (2\pi \text{ rad/cycle})f$$

So, a frequency of 10,000 cycles per second is equivalent to an angular frequency of $2\pi \times 10,000$ radians per second (or about 62,800 radians per second).

Check Point 9.1: A sinusoidal voltage with peak value 10.0 V oscillates with a period of 1 ms. What is the frequency and angular frequency of the signal?

Just as the voltage can be described in terms of a frequency and angular frequency, so can the current. In fact, for all of the cases we'll examine, the current in the circuit will have the same frequency as the applied voltage.

Check Point 9.2: A sinusoidal current oscillates with a period of 1 ms. What is the frequency and angular frequency of the current?

9.2.2 Strength

Measuring the actual voltage[3] or current in an AC circuit (i.e., where we have applied an AC voltage) is more difficult than in a DC circuit because the AC voltage and current are constantly changing. We can't use the DC voltage scale on the multimeter because the voltage keeps changing.

For example, a typical AC voltage varies from some maximum value[4], say $V_{\max}$, and some minimum value, $V_{\min}$. In other words, it oscillates between $V_{\max}$ and $V_{\min}$.

There are four basic ways that one can describe how the voltage (or current) varies in strength: peak-to-peak value, amplitude, average value, or root-mean-square value. Let's look at each one.

Peak-to-peak value

The **peak-to-peak value** is just the difference between the maximum ($V_{\max}$) and minimum ($V_{\min}$) values ($I_{\max}$ and $I_{\min}$ values for current). For example, if the voltage varies from +2 V and −2 V, the peak-to-peak value would be 4 V.

Amplitude

As in volume I when we discussed oscillations, the **amplitude** is the maximum value *from the middle value.*

What is the middle value?

For sinusoidal oscillations (like the type we are examining here), the middle value is the average value. For our purposes, we'll assume that the middle value is zero. This need not be the case but it makes the equations simpler.

[3] In keeping with standard practice, I will tend to refer to the AC "voltage" as opposed to the AC "potential difference".

[4] From this point on, I will follow the more standard practice of representing the potential difference as simply V instead of ΔV. I could've done this earlier but I wanted to make sure that you recognized that the symbol represented a difference (across an element or circuit). Nonetheless, you need to keep in mind that it is actually a difference between two points and will depend on which two points in the circuit we use.

For example, if the voltage varies from +2 V and −2 V, the middle value would be 0 V (directly between +2 V and −2 V). In that case, the voltage oscillates about zero, with the maximum above zero (V_{max}) having the same absolute value as the minimum below zero (V_{min}) (i.e., $V_{\text{min}} = -V_{\text{max}}$). In this case, the maximum voltage is +2 V and the minimum voltage is −2 V.

If the middle value is zero, what is the amplitude?

If the middle value is zero, the amplitude is simply V_{max} (i.e., 2 V in our example). Since we are assuming for our discussions that the middle value is zero, we will represent the amplitude as V_{max}.[5]

Average value

The voltage is only at the maximum (or peak) value for an instant of each cycle. Wouldn't the average value be more useful?

Contrary to what you might think, the average value of an AC voltage isn't too useful. As mentioned above, the average value of our signals will be zero.[6] After all, the voltage is oscillating. As it does, it spends just as much time exhibiting potentials above zero as it does potentials below zero. If the middle value is zero and the oscillation is sinusoidal, the average is zero.

Indeed, a DC voltmeter essentially tries to find the average voltage. If you try to use a DC voltmeter to measure an AC voltage, you'll typically get a value of zero, especially if the frequency is high.

Root-Mean-Square (RMS) Values

So how do we measure AC voltage?

There are two ways. One way is to use the "AC" option of the multmeter. The "AC" option measures what we call the RMS (for Root-Mean-Square). It is called the RMS because it represents the square root of the average square value.

[5] Sometimes this is written as V_{peak}.

[6] And, even for other types of oscillations, the average will still be around the middle value.

What is meant by "average square value"?

We know from above that the average value is zero because the values are positive as often as they are negative. However, if you square all of the values, you only get positive values. Then, the average of the squared values will not be zero.

The RMS uses the **average** of all of the **squared values** and takes the square root of it.

For example, suppose we have a sinusoidal function that varies from a value of $+A$ to a value of $-A$. The straight average will be zero. However, if you square all of the values, you find that the squared values range from a high of $+A^2$ to a low of zero. There are no negative values.

If you then take the average of all of the squared values, you will find that the average value is equal to $A^2/2$.

The RMS value is then obtained by taking the square root of this result (i.e., $\sqrt{A^2/2}$), which is equal to $A/\sqrt{2}$.

With a sinusoidal voltage of maximum $V_{\rm max}$, this means that the RMS value[7] is

$$V_{\rm rms} = \frac{V_{\rm max}}{\sqrt{2}} \tag{9.1}$$

Example 9.1: The amplitude of a sinusoidal signal is 3 V. What is the RMS value?

Answer 9.1: The amplitude is the maximun value. Since it is sinusoidal, divide by the square root of 2 to get the RMS value (i.e., 2.1 V).

Check Point 9.3: (a) The voltage of a typical AC outlet (in the United States)

[7]Ideally, a multimeter on AC setting will provide the RMS value (as long as the amplitude is not changed). However, at low frequencies, the averaging mechanism won't be able to obtain the full signal and thus it gives either an incorrect reading or a reading that changes. At high frequencies, the averaging mechanism should be independent of frequency but most hand-held voltmeters can not reliably measure the rms voltage (or current) of very high frequency signals either. This limit depends on the quality of the voltmeter. In general, we need to use the oscilloscope for high frequency signals (see section 9.3).

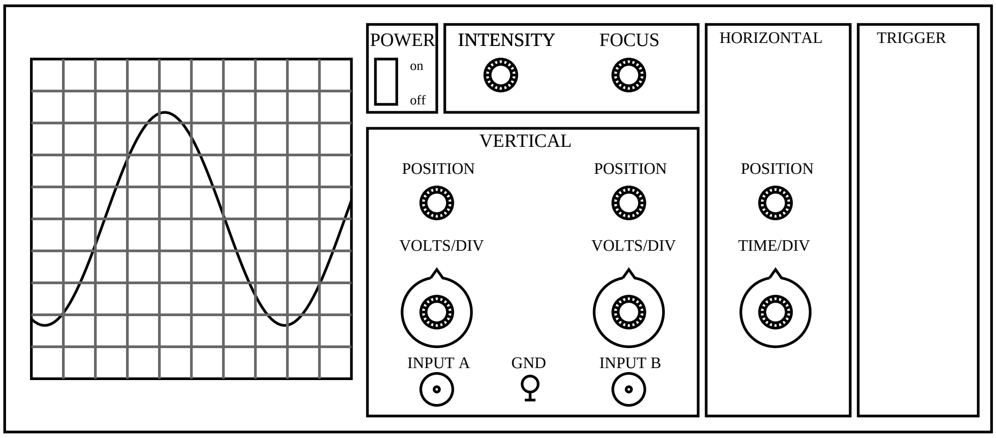

Figure 9.1: A schematic of an oscilloscope. Parts are described in the text.

has an rms value of about 120 V. What is the amplitude of the voltage?
(b) Which is greater: the rms value or the maximum value?
(c) Explain why your answer to (b) makes sense, based on equation 9.1.
(d) Explain why your answer to (b) makes sense, based on what "rms" represents (i.e., the square root of the mean squared value).

9.3 The oscilloscope

As mentioned above, there are two ways to measure an AC voltage. The "AC" scale of a multimeter is one way. That gives the RMS value.

The other way is to use an instrument called an *oscilloscope*.

WHAT IS AN OSCILLOSCOPE?

An oscilloscope is just a fancy voltmeter. An example of an oscillscope is shown in figure 9.1.

At first glance, an oscilloscope looks very complicated because there are many controls on the face of the oscilloscope. Fortunately, the controls break down nicely into six areas. Of the six areas, only three are relevant to our discussion:

1. The display area is the gridded area on the left. This is where the voltage signal is displayed. Unlike a multimeter, which displays just a number, an oscilloscope graphs the voltage as it varies in time. In this case, the figure shows a voltage that is oscillating sinusoidally (like a sine function).

2. The area entitled "VERTICAL" controls the vertical axis of the graph. This axis represents the **voltage** of the source. This area is separated into two parts (A and B) in case the user wants to measure the voltage of two different sources (in which case there would be two plots in the graph area).

3. The area entitled "HORIZONTAL" controls the horizontal axis of the graph. This axis represents **time**.

The way you make a voltage measurement with the oscilloscope is similar to the way you make a voltage measurement with a multimeter. Remember that to measure the voltage of a battery with a *multimeter*, one connects the "COM" (or "−") port of the meter to one end of the battery and the "V" (or "+") port of the meter to the other end.

The *oscilloscope*, like the multimeter, also has two connections to measure the voltage but it differs from the multimeter in two ways.

One way it differs is that the oscilloscope can measure <u>two</u> voltage differences at the same time. As mentioned before, the oscilloscope's "VERTICAL" section has two parts. Each part has a duplicate set of controls. Each set corresponds to a separate input (or "channel").

Why do you need two inputs?

The two inputs are provided in case you want to compare the voltage across one part of the circuit with the voltage across another part of the circuit.

Where are the "+" and "−" ports of the oscilloscope?

That is the other difference between the oscilloscope and a multimeter.

Rather than having two "banana" slots (one for "+" and and one for "−"), there is a separate "+" port for each input and a common "ground" port.[8]

[8] Actually, each input is "co-axial", which means there is an outer conductor wrapped around an inner conductor. This can be converted to the typical dual-input plugs via a "banana to co-axial" converter (or BNC clip). The BNC clip usually has a red port (for "+") and a black port (for "−").

In the figure, the ground port is indicated as "GND" and the two input ports are indicated as "INPUT A" and "INPUT B".

So, to measure the voltage of a battery, you connected the input port to one end of the battery and the ground port to the other side.

How do we use the oscilloscope to measure the voltage across an element in a circuit?

The method is similar to how you would measure the voltage across a battery. Connect the input port to one end of the element and the ground port to the other side.

How do you know what the voltage is if the oscilloscope doesn't give you a number?

To determine the voltage, you need to convert the graphical display to a numerical value. Voltage is measured vertically on the graphical display. Each horizontal line on the display represents a certain amount of voltage. The voltage that corresponds to each horizontal line is given by the "VOLTS/DIV" scale (see figure). Changing the "VOLTS/DIV" scale does not change the voltage, it only changes how much each horizontal line represents.

For example, if in the figure the "VOLTS/DIV" scale was 1 V/division, then the signal being measured in the figure has an amplitude of about 3.3 V (i.e., 6.6 V peak-to-peak).

Check Point 9.4: If the "VOLTS/DIV" scale in figure 9.1 was 0.2 V/division, what is the ampltiude of the signal being measured?

What benefit does the graph have?

The benefit of the graph as opposed to just a number is that it allows us to show how the voltage varies with time (which is the horizontal axis). The time scale can also be chosen. It is this capability that allows us to see phase differences between different signals.

How do we measure two voltages at once?

For the most part, you can treat the two inputs to the oscillscope as two separate "voltmeters", each with their own controls for scale.

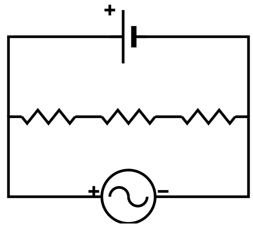

Figure 9.2: A schematic of a circuit containing a battery and three resistors in series, with the voltage across the battery being measured by an oscilloscope.

Important: Since each input shares the same ground ("GND"), you cannot choose two different places in the circuit for "GND." If you do, those two "GND" locations are then connected (within the oscillscope) and you'll short out a portion of the circuit.

Because of the graphing ability, we indicate the oscilloscope in a circuit schematic via a circle with a wave on it (see figure 9.2).

Can we use the oscilloscope to measure the current through the circuit also?

No. The oscilloscope measures voltage, not current.

However, we can still use it to measure current indirectly. In other words, we could measure the voltage across a resistor. Then, assuming we know the resistance of the resistor, we could then use $V = IR$ to calculate the current through the resistor.

Check Point 9.5: Suppose we wanted the oscillscope to measure both the voltage across the battery and the voltage across one of the resistors. Which of the three resistors can the second channel measure and why? Remember that the oscilloscope only has one ground. So, the "−" side of each oscilloscope channel must be connected to the same location.

9.4 Signal Generators

If batteries produce DC voltage, how do we get AC voltage?

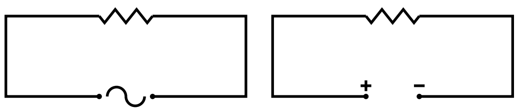

Figure 9.3: [left] A schematic of an AC voltage source connected to a resistor. [right] A schematic of a DC voltage source connected to a resistor.

If we are going to study the effect of AC voltage on circuits, we need a reliable, adjustable source of AC voltage. The source should be able to provide a variety of frequencies and amplitudes.

Such a device is called a signal generator[9]. In a sense, a signal generator is just a very fancy battery.

In a circuit schematic, an AC voltage source (such as that produced by a signal generator) is indicated by a wavy line (see left schematic in figure 9.3).[10]

The "+" and "−" outputs of the signal generators is somewhat analogous to the "+" and "−" sides of a battery except that the voltage oscillates.

Note: With many signal generators, the "−" port is actually connected to the building ground. If the ground of the oscilloscope is likewise connected to building ground then the ground of the signal generator must be connected to the same part of the circuit as the ground of the oscillscope. Otherwise, you'll short out part of the circuit.

Check Point 9.6: A signal generator is set to produce a sinusoidal voltage with a period of 1 s. If a light bulb is connected to the signal generator, with what frequency does the light bulb blink?†

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[9]There are many different types of signal generators available, ranging from those that only provide certain types of oscillations (e.g., sinusoidal) at only certain frequencies to those that allow you to specify any arbitrary signal. These latter generators usually go by the name "Arbitrary Waveform Generators". There are also special generators for tuning radios, and measuring various characteristics of circuits.

[10]For reference, if a power supply was providing DC (instead of AC), we would use something similar but with the + and − sides indicated (see right schematic in figure 9.3). This would be used instead of the battery symbol.

9.5 Relationship Between Voltage and Current

FOR DC, THE RELATIONSHIP BETWEEN VOLTAGE AND CURRENT WAS $\Delta V = IR$. WHAT IS THE RELATIONSHIP FOR AC?

It is still $\Delta V = IR$. However, from now on, we'll write the potential difference as V and so the relationship will be written as $V = IR$. Just keep in mind that V is a potential difference.

IN SECTION 9.2.2, IT WAS MENTIONED THAT THERE ARE SEVERAL DIFFERENT WAYS ONE CAN DESCRIBE HOW THE VOLTAGE (OR CURRENT) VARIES IN STRENGTH. WHICH WAY DO WE USE IN $V = IR$?

It doesn't matter. However, whatever way you use for V, you must also use for I. For example, if you use the maximum value for voltage (V_{max}), you must also use the maximum value for current (I_{max}):

$$V_{\text{max}} = I_{\text{max}} R. \tag{9.2}$$

Or, if you use the RMS value for voltage (V_{rms}), you must also use the RMS value for current (I_{rms}):

$$V_{\text{rms}} = I_{\text{rms}} R. \tag{9.3}$$

Check Point 9.7: (a) If we use $V = IR$ with the resistance and the RMS current, does V correspond to the RMS voltage or the voltage amplitude?
(b) The RMS current through a 20-Ω resistor is 2.0 mA. What is the RMS voltage across the resistor?

9.6 Power in an AC Circuit

For DC, the relationship between power, voltage and current was $P = IV$ (equation 6.4).

WHAT IS THE RELATIONSHIP FOR AC?

We can use the same relationship but only if we use the RMS values.

Why?

The reason we use the RMS values is because properties like the energy consumed by the circuit are more closely related to the RMS values than the peak values.

For example, we know from equation (6.4) that, at any given time, the power (i.e., the rate at which energy is dissipated) is related to the voltage and current at that time via $P = IV$. However, the *average* power is what we are usually interested in and the average power equals the product of the RMS values.

Wouldn't the average power equal the product of the average values?

No. Consider, for example, the case when the average values of voltage and current are zero (which we are assuming is the case for the situations we are examining). Even though the average values are zero, when applied to a light bulb the light bulb still lights. Consequently, there is still energy being consumed by the circuit and so the power must not be zero.

Can we use the product of the maximum values?

No. That will only give you the power valid at the time when the voltage and current are a maximum (i.e., the maximum power). Most of the time, the power is less than this.

So, since we are interested in an *average* power consumption, not the maximum power consumption, it turns out that we use the RMS values:

$$P_{\text{avg}} = \left\{ \begin{array}{l} I_{\text{rms}} V_{\text{rms}} \\ V_{\text{rms}}^2 / R \\ I_{\text{rms}}^2 R \end{array} \right\} \tag{9.4}$$

All three ways will give the same answer. The first one listed is based on equation 6.4 ($P = IV$) with the RMS values used. For the others, I used equation 9.3

$$V_{\text{rms}} = I_{\text{rms}} R$$

to replace I_{rms} or V_{rms}.

Example 9.2: The average power dissipated in particular circuit is 55 W.

Assuming a typical AC rms voltage of 1 V (typical in the U.S.), find the RMS value of the AC current through the circuit.

Answer 9.2: Use $P_{\text{avg}} = I_{\text{rms}} V_{\text{rms}}$, with $P_{\text{avg}} = 55$ W and $V_{\text{rms}} = 1$ V. Solve to get an rms current of 0.46 A.

Check Point 9.8: The average power dissipated in a stereo speaker is 55 W. Assuming that the speaker can be treated as a resistor with 4.0-Ω resistance, find:
(a) The rms value of the AC voltage applied to the speaker and the rms value of the AC current through the speaker.†
(b) The peak value of the AC voltage applied to the speaker and the peak value of the AC current through the speaker (i.e., find the amplitudes of the voltage and current).

Review

You should now be able to do the following:

- Describe an oscillating voltage or current in terms of maximum voltage V_{max}, maximum current I_{max}, frequency f, and angular frequency ω, where $\omega = 2\pi f = 2\pi/T$.
- Use a signal generator to generate an oscillating voltage.
- Describe the strength of an oscillating voltage and current in terms of the rms (or root-mean-squared) value, e.g.,

$$V_{\text{rms}} = \frac{V_{\text{max}}}{\sqrt{2}}$$

  and use it to calculate the average power consumed by a circuit, e.g.,

$$P_{\text{avg}} = V_{\text{rms}} I_{\text{rms}}.$$