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## 26. Simple Harmonic Motion

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26.1 *In figure 26.1, five points are plotted on a circle, indicated as a, b, c, d and e, indicating the position of an object at five times, at equal time intervals.*

*(a) Is the object undergoing uniform circular motion (i.e., constant speed along a circular path)? How do you know?*

*(b) Consider an object in a straight line, with positions marked by a', b', c', d' and e'. Is that object moving at a constant speed? How do you know?*

*(a) yes, because the points are equally spaced around the circle, (b) no, because the points are not equally spaced along the horizontal line*

26.2 *Suppose  $\hat{x}$  is toward the right with  $s_x = 0$  at the center of the circle in figure 26.1. What is the sign of  $s_x$  when the object is at point d?*

Negative, since the object is to the left of the center of the circle

26.3 *Suppose  $\hat{x}$  is toward the right with zero at the center of the circle in figure 26.2, with the object moving counter-clockwise around the circle. What is the sign of  $v_x$  when the object is at point d?*

Negative, since the object is moving toward the left

26.4 *When the angular position is 300 degrees, what is the sign of  $a_x$ ? Explain.*

Negative because the object is moving toward the right and slowing down (acceleration opposite velocity)

26.5 *Suppose the radius of the circle in figure 26.1 is 3 cm.*

*(a) What is the value of  $s_x$  when the object is at point e?*

*(b) What is the value of  $s_y$  when the object is at point e?*

*(c) Is it possible for  $s_y$  to equal 4 cm (assuming the object is constrained to move on the circle)? Why or why not?*

(a)  $-3$  cm, (b)  $0$ , (c) no, the largest it can be is  $3$  cm

26.6 *Suppose the radius of the circle in figure 26.2 is  $3$  cm and the object is moving around the circle with a period of  $2$  seconds.*

(a) *What is the speed  $v_t$  of the object as it moves around the circle?*

(b) *What is the value of  $s_x$  when the object is at point  $e$ ?*

(c) *What is the value of  $v_x$  when the object is at point  $e$ ?*

(d) *Is there ever a time when  $v_x$  equals twice the value found in part (a)? Why or why not? Assume the object is constrained to move on the circle.*

(a)  $9.4$  cm/s (b)  $-3$  cm, (c) zero, (d) no, the most it can ever be is the speed in (a)

26.7 *Suppose we have an object undergoing counter-clockwise (looking down) uniform circular motion as in figure 26.2 of radius  $2$  m and a period of  $3$  seconds. Explain your reasoning for each answer.*

(a) *What is the maximum value of  $a_x$ ?*

(b) *What is the value of  $a_x$  at point  $e$ ?*

(c) *What is the value of  $a_x$  at point  $c$ ?*

$8.8$  m/s<sup>2</sup>, (b)  $8.8$  m/s<sup>2</sup>, (c) zero

26.8 *An object moves in a circle of radius  $5$  cm. When the angular position is  $210^\circ$ , what is the  $x$  component of the position?*

$-4.33$  cm

26.9 *An object moves in a circle of radius  $5$  cm. When the angular position is  $220^\circ$ , what is the  $x$  component of the position? Use a calculator and check your value using the graph in figure 26.4.*

$-3.83$  cm

26.10 *An object moves in a circle of radius  $5$  cm with a period of  $2$  s. When the angular position is  $220^\circ$ , what is the  $x$  component of the velocity? Use a calculator and check your values using the graphs in figure 26.4.*

$10.1$  cm/s (the maximum velocity is  $15.7$  cm/s)

- 26.11 *An object moves in a circle of radius 5 cm with a period of 2 s. When the angular position is  $220^\circ$ , what is  $a_x$ ?*

37.8 cm/s<sup>2</sup> (the maximum acceleration is 49.35 cm/s<sup>2</sup>)

- 26.12 *Suppose we have an object undergoing counter-clockwise (looking down) uniform circular motion as in figure 26.2 of radius 2 m and a period of 3 seconds.*

(a) *What is the value of  $\omega$ ?*

(b) *What is the maximum value of  $v_x$ ?*

(c) *What is the maximum value of  $a_x$ ?*

(d) *What is the value of  $v_x$  when the angular position is  $220^\circ$ ?*

(e) *What is the value of  $a_x$  when the angular position is  $220^\circ$ ?*

(a) 2.1 rad/s (b) 4.2 m/s (c) 8.8 m/s<sup>2</sup> (d) 2.69 m/s (e) 6.72 m/s<sup>2</sup>

- 26.13 *Suppose an object moves at constant speed in a circle of radius 3 cm with a period of 4 seconds. If the object starts at  $\theta = 0$ , how long does it take until  $\theta = 3\pi/2$  radians?*

3 s

- 26.14 *An object undergoes simple harmonic motion with a period of 3 seconds.*

(a) *After 1.5 seconds, what fraction of a cycle has the object experienced?*

(b) *If it starts at a position all the way on the right, where is it after 1.5 seconds?*

(a) One-half of a cycle, (b) all the way on the left

- 26.15 *An object undergoes simple harmonic motion with a period of 3 seconds and an amplitude of 2 cm. If it starts at a position  $s_{\text{shm}} = 2$  cm, what is the value of  $s_{\text{shm}}$  after 1.5 seconds?*

-2 cm

- 26.16 *A block is released at rest from a point 4 cm from equilibrium. It then undergoes simple harmonic motion with an amplitude of 4 cm and a period of 3 seconds.*

(a) *What is the maximum distance the object will ever be from the equilibrium point?*

(b) After one second of motion, has the object passed the equilibrium position? Explain.

(c) One second after release, is it moving away from its initial position or toward? Is it moving away from the equilibrium position or toward? Explain.

(d) One second after release, is the object speeding up or slowing down? Explain.

(a) 4 cm, (b) yes, since 1 second is more than a quarter of the way through the cycle, (c) both are away since it is to the left of the equilibrium position and not yet half-way through the cycle, (d) slowing down, since it is past the equilibrium position starting to approach the turn-around point

26.17 An object undergoes simple harmonic motion with a period of 3 seconds.

(a) What is the frequency of the motion?

(b) What is the angular frequency of the motion?

(c) If the angular position starts at  $\pi$  radians, express the angular position of the object, in radians, after 1 second.

(a)  $1/3$  Hz; (b)  $2\pi/3$  rad/s; (c)  $5\pi/3$

26.18 A block undergoes simple harmonic motion with an initial phase of  $45^\circ$ , a period of 3 seconds and an amplitude of 4 cm.

(a) What is the initial position of the motion?

(b) What is the initial velocity of the motion?

(c) What is the initial acceleration of the motion?

(a) 2.83 cm, (b) 5.92 cm/s, (c)  $12.4 \text{ cm/s}^2$

26.19 Suppose the spring in figure 26.5 has a spring constant  $k$  of 50 N/m. A 500-g object, attached to the spring, is released from rest at a point 4 cm from equilibrium.

(a) What is the force of the spring on the object when the object is 4 cm from equilibrium?

(b) What is the force of the spring on the object when the object is right at the equilibrium position?

(a) 2 N, (b) zero

26.20 Show how one can obtain equation 26.15 by combining equation 26.11

$$s_{\text{shm}} = A \cos(\omega\Delta t + \theta_i)$$

and equation 26.13

$$a_{\text{shm}} = -\omega^2 A \cos(\omega\Delta t + \theta_i).$$

Use the first expression to replace  $A \cos(\omega\Delta t + \theta_i)$  in the second expression with  $s_{\text{shm}}$ .

26.21 Suppose we have a spring that follows Hooke's law, where  $k$  is 50 N/m.

(a) What is the maximum force exerted by the spring on a 50-g object undergoing an oscillation with an amplitude of 8 cm?

(b) What is the maximum acceleration attained by the object?

(c) What is the period of the motion?

(a) 4 N, (b) 80 m/s<sup>2</sup>, (c) 0.2 s