Section A

1. b (for an object moving in a circle, the net force must be directed toward the center of the circle)

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3. c (use the radius of the path the object, in this case the moon, is following)

4. a (Since the radius is 50 m, then one radian would correspond to an arc length of 50 m. An arc length of 10 m is only one fifth of that.)

5. d (You, and the Earth, make one revolution each day. There are $2\pi$ radians in one revolution.)

6. b (From Newton's second law, the net torque is equal to the rotational inertia times the angular acceleration; so, the correct choice must correspond to the situation where the torque is maximum. Since the torque is defined as the tangential force times the distance to the axis, the correct answer is the one that has the force directed tangentially "around the axis" (the hinges in this case).)

7. a (The net torque must be zero; torque is the tangential force (the gravitational force on the masses) times the distance from the pivot; since the 50-g mass is 30 cm from the pivot, the 150-g mass must be one-third as far and on the opposite side)

8. b (In this case, the stick is off-center so it is like having a 50 g object placed 10 cm from the pivot; to balance that, you'd need to have the 100 g object 5 cm from the pivot on the opposite side)

9. e (There are a couple ways to do this. The simplest way is to recognize that the maximum velocity is equal to $2\pi A/T$, where $A$ is the amplitude and $T$ is the period. The period is is independent of the amplitude so if the amplitude doubles then so does the maximum velocity)

10. e (when the object has reached its greatest speed, it is no longer speeding up and it has not yet started to slow down)

11. a (The air rotates with the earth and, as it is pulled toward the center of the hurricane, it spins faster in the same direction to conserve angular momentum; this direction appears counter-clockwise in the northern hemisphere)

12. c (Since they have the same shape, as they roll down the incline, both have the same fraction of their kinetic energy involved in rotational kinetic energy vs. translation kinetic energy)

Section B

1. (a) I forgot to provide the radius of the Earth ($6.37 \times 10^6$ m; although I specify it in problem 3) and the mass of the Earth ($5.98 \times 10^24$ kg; although you get to calculate it in problem 2), which allow you to use the universal law of gravitation ($F_g = Gm_1m_2/r^2$) with $r$ being the distance from the center of the astronaut to the center of the Earth. Add the radius of the Earth to 350,000 m to get $r$ then solve to get a force of 620 N.

   (b) For this, you can repeat the calculation from part (a) with $r$ equal to the radius of the Earth or you can simply use $mg$, where $g$ is 9.8 N/kg. I get a force of 690 N.

2. For the moon to move in uniform circular motion, it must have a speed equal to $2\pi r/T$. Converting the period into seconds ($2.36 \times 10^6$ s), I get a speed of $1.02 \times 10^3$ m/s.

   Furthermore, the moon must be undergoing an acceleration (changing directions) at a rate equal to $2\pi v/T$ and directed toward the center of the circle. Plugging in the speed and the period (converted to seconds), I get an acceleration of 0.0146 m/s$^2$ (toward the Earth).

   That means there must be a force acting on the moon equal to $2.72 \times 10^{-3}$ m/s$^2$ times the moon's mass. Even though we don't have the mass, we can still continue (because the mass will cancel out later).

   In this case, that force is being provided by the gravitational attraction between the earth and the moon. Using the universal law of gravitation and setting the gravitational force equal to the moon's mass times the acceleration, we have

$$G\frac{m_{\text{earth}}m_{\text{moon}}}{r^2} = m_{\text{moon}}(2.72 \times 10^{-3}\ \text{m/s}^2)$$

   Here the mass of the moon cancels on both sides. Plugging in $3.88 \times 10^8$ m for the radius of the circle (i.e., radius of the moon's orbit) and $6.67 \times 10^{-11}$ Nm$^2$/kg$^2$ for the gravitational constant, I get a mass of $6.04 \times 10^{24}$ kg. This is slightly more than the actual value because the center of the moon's orbit is not coincident with the center of the earth.

3. (a) The speed is equal to the circumference divided by how long it takes to go all the way around:

$$\begin{aligned} v &= (2\pi r)/T \\ &= 2\pi(6.37 \times 10^6 \text{ m})/(24 \times 3600 \text{ s}) \end{aligned}$$

which gives a speed of 463.2 m/s.

(b) The acceleration of an object in circular motion is directed toward the center of the circle and has a magnitude equal to:

$$\begin{aligned} a_c &= (2\pi v)/T \\ &= 2\pi(463.2 \text{ m/s})/(24 \times 3600 \text{ s}) \end{aligned}$$

which gives an acceleration of 0.0337 m/s$^2$ toward the center of the Earth (center of circular path in this case).

(c) From Newton's second law, the net force on the student must equal the product of the student's mass and acceleration:

$$\begin{aligned} F_{\text{net}} &= ma \\ &= (70 \text{ kg})(0.0337 \text{ m/s}^2) \end{aligned}$$

which gives a net force of 2.36 N (toward the center of the circle).

(d) The gravitational force is equal to $mg$ or 686 N. If that is equal to the net force (i.e., that is the only force acting) then there is no normal force at all. To find the velocity at which this would be true, we work backwards. From Newton's second law, the acceleration would have to be $F_{\text{net}}/m$ or 9.8 m/s$^2$. Knowing the acceleration, we can use $a = v^2/r$ to get the velocity (since $r$ is known). I get a speed of $7.9 \times 10^3$ m/s. This is the speed an object must travel (at the earth's surface) to be "in orbit" (i.e., without falling down to the ground).

4. (a) This part describes a situation where information about the motion is provided (constant angular acceleration, initial and final rotation rates) and asks about another aspect of the motion (angular displacement). This requires us to use the definitions of motion variables or, since the acceleration is constant in this case,

the relationships that are valid for constant acceleration:

$$\begin{aligned} \vec{v}_{\text{avg}} &= \tfrac{1}{2}(\vec{v}_{\text{i}} + \vec{v}_{\text{f}}) \\ \Delta\vec{s} &= \vec{v}_{\text{i}}\Delta t + \tfrac{1}{2}\vec{a}(\Delta t)^2 \end{aligned}$$

You can use either one to solve the problem. However, since the problem describes rotation, we should use rotational variables:

$$\begin{aligned} \omega_{\text{avg}} &= \tfrac{1}{2}(\omega_{\text{i}} + \omega_{\text{f}}) \\ \Delta\theta &= \omega_{\text{i}}\Delta t + \tfrac{1}{2}\alpha(\Delta t)^2 \end{aligned}$$

This means that we can either determine the average angular velocity (5 rev/s, since it starts at 10 rev/s and ends at zero) then use the definition of average angular velocity ($\omega_{\text{avg}} = \Delta\theta/\Delta t$) to solve for the displacement:

$$\begin{aligned} \Delta\theta &= \omega\Delta t \\ &= (5 \text{ rev/s})(15 \text{ s}) \\ &= 75 \text{ rev} \end{aligned}$$

or we can first determine the angular acceleration ($-0.67$ rev/s$^2$, since it takes 15 s to go from 10 rev/s down to zero) and then solve for the displacement as follows:

$$\begin{aligned} \Delta\theta &= \omega_{\text{i}}\Delta t + \tfrac{1}{2}\alpha(\Delta t)^2 \\ &= (10 \text{ rev/s})(15 \text{ s}) + \tfrac{1}{2}(-0.67 \text{ rev/s}^2)(15 \text{ s})^2 \\ &= (150 \text{ rev}) - (0.33 \text{ rev/s}^2)(225 \text{ s}^2) \\ &= 75 \text{ rev} \end{aligned}$$

which gives the same answer. Notice that the acceleration is negative because the object is slowing down and I used a positive value for the initial angular velocity.

(b) Since this question asks about work (which is related to force) and the problem gives information about the motion then, at some point, you will need use a version of Newton's second law (either $\vec{F}_{\text{net}} = m\vec{a}$ or $W_{\text{net}} = \Delta K$).

Either way, since we are applying Newton's second law to rotation, we will first need to determine the rotational inertia $I$. Rotational inertia is defined as $mr^2$ but for a disk the rotational inertia is $\frac{1}{2}MR^2$. It isn't $MR^2$ because the mass is distributed between a distance zero to a distance $R$ from the rotation axis (i.e., the entire mass is not at a distance $R$ from the rotation axis). Plugging in the mass (4 kg) and radius (2 m), we get a rotational inertia of 8 kg·m$^2$.

Another thing to watch out for when using the rotational inertia is that any angles must be given in radians. For example, we can't use $\omega$ equal to 10 rev/s. Instead, we must convert this into radians. Since one revolution is $2\pi$ radians, that means that the disk is initially rotating at $20\pi$ rad/s.

As mentioned before, we will need to use either $\vec{F}_{\text{net}} = m\vec{a}$ or $W_{\text{net}} = \Delta K$. Which you use is up to you. For example, you could use the definition of work, which is torque times angular displacement (for rotation). That means you'll first need to calculate the torque via $\vec{F}_{\text{net}} = m\vec{a}$ (which, for rotation, is $\tau = I\alpha$). And that means you'll first have to determine the angular acceleration *alpha* (which we might have determined in part (a) but we'd first have to convert to radians per second squared).

An alternative way, and perhaps simpler way, is to use $W_{\text{net}} = \Delta K$. To determine the work this way, first determine the change in kinetic energy. Kinetic energy is defined as $\frac{1}{2}mv^2$ but for rotation it is $\frac{1}{2}I\omega^2$. Plugging in the initial angular velocity (in rad/s) and the rotational inertia, I get:

$$\begin{aligned} K_{\text{rot,i}} &= \tfrac{1}{2}I(\omega)^2 \\ &= \tfrac{1}{2}(8\ \text{kg}\cdot\text{m}^2)(20\pi\ \text{rad/s}^2)^2 \\ &= 1600\pi^2\ \text{J} \end{aligned}$$

which is $1.58 \times 10^4$ J. The final rotational kinetic energy is zero, so the change in rotational kinetic energy is $-1.58 \times 10^4$ J. This must also be equal to the work done on the disk.

(c) The work done is equal to the force times the displacement (in the direction of the force). For rotation, this means the work is the torque times the angular displacement. Since we know the work and we know the angular displacement (75 rev, from before), we can get the torque.

However, as in part (b), we must use the displacement in radians in order for the units to work out. Again, as before, there are $2\pi$ radians in every revolution. So, the angular displacement is $150\pi$ radians.

The torque, then, is the work divided by the angular displacement. I get 33.5 N·m.

5. There are three forces acting on the plank. One is the gravitational force on the plank (15 kg times 9.8 N/kg = 147 N), acting at the center of gravity (which, since the plank is uniform, is in the middle of the plank, 1.5 m from each end). Another is the force exerted by the weight of the child (30 kg times 9.8 N/kg = 294 N), acting at the location of the child (unknown). The third is the force of the pivot (unknown), acting at a point 1.0 m from one end.

   We have two unknowns: the location of the child and the force of the pivot. We can get the force of the pivot by balancing the upward and downward forces. However, we aren't asked for the force of the pivot – we are asked for the location of the child. To get that, balance the torques. Since we don't know the force of the pivot, set the rotation axis at that location, which means that torque of the pivot would be zero about that axis.

   The torque due to the gravitational force on the plank about that axis is the force, 147 N, times the distance from where the force is acting to the axis of rotation. Since we've set the axis of rotation at the pivot, the distance is 0.5 m (from the middle of the plank to the pivot). That gives a torque of 73.5 Nm.

   The torque due to the child is the force, 294 N, times the unknown distance. However, this torque must balance the other torque. Thus, we have

   $$(294\ \text{N})r = 73.5\ \text{N} \cdot \text{m}$$

   and, solving for $r$, we get a distance of 0.25 m. This is from the axis, remember, so since we set the axis as the pivot (for this problem), the child must be placed 0.25 m from the pivot or 0.75 from the end of the plank (on the short side of the pivot).

6. The way to solve this is to first consider the torques acting on the ladder. If we set the axis at the bottom, the torque exerted by the

floor (about that axis) is zero. That is convenient since we don't know (or need to know what that force is). The only other torques acting on the ladder is the one by gravity and the one by the wall. These should balance out (assuming the ladder isn't slipping).

To find the torque due to gravity, first identify the force of gravity. In this case, the force is the mass (2 kg) times 9.8 N/kg. That means the force is 19.6 N. To find the torque, we need the tangential part of this. Multiply by the cosine of 60 degrees to get that (since the force is directed 60 degrees from the tangential direction; draw a picture). That gives a tangential force of 9.8 N. Multiply this by the distance to the axis (2 m; since the center of gravity is at the center of the ladder) to get a torque of 19.6 N·m (acting clockwise around the chosen axis).

To be balanced, the wall must be provided a counter torque equal to 19.6 N·m acting counter-clockwise. Since the force of the wall is 4 m from the chosen axis, that means it must be exerting a tangential force equal to $\tau/r$ or 4.9 N. This is only the tangential component of the force. To find the total force, we divide by the cosine of 30 degrees (since the force of the wall is directed 30 degrees from the tangential direction; again draw a picture). This gives a force of 5.66 N.

7. (a) Since it is released 2 cm from the equilibrium position, it won't get more than 2 cm away from equilibrium. The amplitude of the motion is 2 cm.

   (b) The net force on the object at equilibrium is zero. That is why it is called the equilibrium position – an object placed there at rest will stay at rest at that location.

   (c) Assuming an ideal spring, where the force of the spring is proportional to the displacement, the net force on the object at its maxium displacement is equal to $kx$, where $k$ is the spring constant (spring strength) and $x$ is the displacement (2 cm in this case). Since hanging a 20-g weight (with gravitational force equal to 0.02 kg times 9.8 N/kg) stretches the spring 3 cm, we have that

$$\begin{aligned} k &= F/x \\ &= (0.02 \text{ kg}) \times (9.8 \text{ N/kg})/(0.03 \text{ m}) \end{aligned}$$

   which gives a spring constant of 6.53 N/m.

When stretched an additional 2 cm (which the spring is when the object is 2 cm below the equilibrium position), the spring force is (6.53 N/m)$\times$(0.02 m) more. This means the net force there is 0.13 N.

(d) There are several ways to find the acceleration. The easiest, perhaps, is to use Newton's second law. Since we already know the net force at the lowest point (see part c), divide by the mass to find the acceleration. I get an acceleration of 6.53 m/s$^2$.

(e) To find the period, we can use the fact that the maximum acceleration must equal $4\pi^2 A/T^2$, where $A$ is the amplitude. Solving for $T$, I get a period of 0.358 s.

(f) The maximum speed (which occurs when the object is passing the equilibrium point) is equal to $2\pi A/T$. Plug in to get 0.361 m/s.