

Section A

1. e (the total momentum is conserved, not the velocity; the best way to approach this is to recognize that we don't even know the *direction* the blocks will move. They will move toward the left after the collision if block B's momentum to the left is greater than block A's momentum toward the right; otherwise the blocks will move toward the right after the collision.)
2. a (since the masses are the same, we can at least recognize that the blocks will move toward the left after the collision; if they move at 5 m/s, then block A changes from 10 m/s right to 5 m/s left; that is the same, but opposite, change as block B's 20 m/s left to 5 m/s left)
3. c (total energy is conserved and has to be converted to another type of energy; gravitational energy doesn't change, or goes down, when it is in contact with the ground, and no light is produced)
4. a (since the velocities must be the same, the difference in energy must only be due to the difference in mass; the kinetic energy is proportional to the mass)
5. d (the only way for the kinetic energy to be zero is if they are both stationary)
6. b (motion has to be vertical for gravity to do work on the object)
7. b (initial velocity is 1.5 times what it was before; Since the kinetic energy is defined as $\frac{1}{2}mv^2$ that means the kinetic energy is proportional to the square of the speed. If the speed increases by a factor of 1.5, the energy increases by a factor of $(1.5)^2$. Consequently, the work done to stop the truck must be 1.5^2 times greater also.)
8. e (by doubling the speed, we quadruple the kinetic energy; the force of friction would be the same but with four times as much kinetic energy one would need to slide four times as far to produce four times as much work)
9. a (if only conservative forces were acting, like gravity, then the kinetic energy would have to return to whatever value it had initially since

there is no difference in gravitational energy if an object doesn't change its position)

Section B

1. (a) Apply conservation of energy, since direction of motion is not needed and path is curved (which makes determining the normal force difficult). As the first block slides down the ramp, it gains kinetic energy (i.e., it speeds up). Since there is no loss of energy from friction, the gain in kinetic energy must equal the loss in gravitational energy. The change in gravitational energy is $mg\Delta y$, which equals $(0.040 \text{ kg})(9.8 \text{ N/kg})(-1.00 \text{ m})$ or -0.392 J . The negative means that gravitational energy was lost. In the process, kinetic energy increases by the amount that gravitational energy decreased. This means that the block gained 0.392 J of kinetic energy. Since it started at rest, the block's kinetic energy at the bottom must be 0.392 J .

Since $K = \frac{1}{2}mv^2$, one can solve for the block's speed to get 4.427 m/s .

- (b) Momentum is conserved. In this case, the momentum before the collision is known. The first block is moving at 4.427 m/s . Multiply by its mass (0.040 kg) to get a momentum of $0.177 \text{ kg}\cdot\text{m/s}$ toward the right. The initial momentum of the second block is zero (since it is at rest prior to the collision). Thus, the total momentum prior to the collision is $0.177 \text{ kg}\cdot\text{m/s}$ toward the right. Since momentum is conserved, that must also be the total momentum just after the collision.

$$\begin{aligned} 0.177 \text{ kg}\cdot\text{m/s} &= m_1v_{1,f} + m_2v_{2,f} \\ &= (0.040 \text{ kg})v_{1,f} + (0.070 \text{ kg})v_{2,f} \end{aligned}$$

Without more information, we cannot tell how much momentum is associated with one block vs. the other. To get that, we use the knowledge that the collision is elastic. That means that after the collision block 2 must be moving 4.427 m/s faster than block 1

(since the difference in speed must equal the same as what it was before the collision)ⁱ.

$$4.427 \text{ m/s} = v_{2,f} - v_{1,f}.$$

Rearranging the momentum equation to solve for $v_{1,f}$ (the variable we don't really need and thus want to get rid of), we get

$$v_{1,f} = \frac{0.177 \text{ kg} \cdot \text{m/s} - (0.070 \text{ kg})v_{2,f}}{(0.040 \text{ kg})}$$

and plugging that into the second equation, we get

$$\begin{aligned} 4.427 \text{ m/s} &= v_{2,f} - \frac{0.177 \text{ kg} \cdot \text{m/s} - (0.070 \text{ kg})v_{2,f}}{(0.040 \text{ kg})} \\ &= v_{2,f} \left[1 + \frac{0.070 \text{ kg}}{0.040 \text{ kg}} \right] - \left[\frac{0.177 \text{ kg} \cdot \text{m/s}}{0.040 \text{ kg}} \right] \\ &= 2.75v_{2,f} - 4.427 \text{ m/s} \end{aligned}$$

which gives us a final speed of block 2 of 3.220 m/s.

Note: Since we know the relative velocity is 4.427 m/s, that means that block 1 (after the collision) must be moving back toward the left at $(4.427 \text{ m/s} - 3.220 \text{ m/s})$ or 1.207 m/s. However, we aren't asked for that.

- (c) Since block 2 has a speed of 3.220 m/s, its kinetic energy is

$$\begin{aligned} K &= \frac{1}{2}mv^2 \\ &= \frac{1}{2}(0.070 \text{ kg})(3.220 \text{ m/s})^2 \\ &= 0.363 \text{ J} \end{aligned}$$

Note: If you calculate the kinetic energy of the first block after the collision, you'll find that the rest of the kinetic energy is associated with that block (since the collision is elastic).

- (d) To get over the second ramp, the second block must have a kinetic energy greater than or equal to the change in gravitational energy as it rises over the second hill. The change in gravitational

ⁱThis also means that the kinetic energy is conserved but using the definition of kinetic energy just makes for messy math.

energy is $mg\Delta y$, which equals $(0.070 \text{ kg})(9.8 \text{ N/kg})(-0.50 \text{ m})$ or -0.343 J . Consequently, block 2 needs to have a kinetic energy greater than or equal to 0.343 J immediately after the collision in order to make it over the second hill.

From part (c), we know that the second block initially has a kinetic energy equal to 0.363 J . Since this is greater than 0.343 J , it will make it over the hill.

2. (a) Yes. The forces are equal and opposite due to Newton's third law and the times are the same, so $\vec{F}\Delta t$ exerted on one object must be equal and opposite to $\vec{F}\Delta t$ exerted on the other object. From Newton's second law, $\vec{F}\Delta t = \Delta\vec{p}$, this means that \vec{p} of one object must be equal and opposite to \vec{p} of the other (where \vec{p} is the momentum $m\vec{v}$).
 - (b) Kinetic energy may or may not be conserved. It is only conserved if the collision is elastic. If it is not elastic, energy goes into deforming the object but we never get that energy back into kinetic energy. Instead the energy goes into heat or some other kind of energy.
 - (c) Yes, since the total amount of energy is always conserved. We just have to make sure we keep track of all the different types.
3. This problem is a good candidate for the work-kinetic energy formulation of Newton's second law (or conservation of energy). This is because there are two forces acting on the gymnast, only one of which does any work on it. One is due to the high bar and the other is due to the Earth (gravitational). The first is toward the bar and, as such, is perpendicular to the motion and thus only acts to change the direction, not speed it up. The only force "doing work" on the gymnast is the gravitational force mg .

When applying conservation of energy, just compare the two positions of interest. In this case, it is when the gymnast is at the top (momentarily at rest) and when the gymnast is at the bottom (where we are trying to determine the speed).

Since the gravitational force is vertical, I only need the vertical displacement between those two positions. In this case, that distance is

1.8 m. Multiply these together to get the work done by gravity: $m(9.8 \text{ N/kg})(1.8 \text{ m})$.

This work must equal the change in kinetic energy: $\frac{1}{2}mv^2$. Set these equal to each other and divide by m to get $(9.8 \text{ N/kg}) \times (1.8 \text{ m}) = \frac{1}{2}v^2$. Solve for v to get a velocity of 5.9 m/s.

You could also do this using the gravitational energy terminology instead of the work terminology. The answer would be the same.

4. (a) The student should throw the shoe away from shore. That requires that the student exerts a force directed away from shore on the shoe. Due to Newton's third law, the shoe would exert an opposite force on the student, pushing the student toward shore.
- (b) From the definition of momentum, the shoe obtains a momentum of mv equal to $(1 \text{ kg})(20 \text{ m/s})$, which is 20 kg m/s away from shore. Due to conservation of momentum, the student must gain an equal amount but in the opposite direction. That means the student's mv must equal 20 kg m/s toward shore. Divide by the mass of the student (60 kg) to get the velocity of the student (0.33 m/s toward shore).