

Section A

1. a (both maintain a constant velocity; with the top one going slower)
2. b (since downward is positive, that means the initial velocity is negative and the acceleration is positive)
3. c (the bullet fired downward has a non-zero vertical velocity whereas the bullet fired horizontally has a zero vertical velocity) [note: an earlier version of these solutions had the incorrect answer to this question]
4. d (the horizontal velocity is the same as what it was on the table)
5. d (since there is no horizontal acceleration during free fall, there is no change in the horizontal velocity)
6. a (the vertical velocity is the same as what it was on the table, where it wasn't moving vertically)
7. d (it ends 10 m south of the initial position)
8. b (it must be greater than either 5, the eastward component, or 10 m, the southward component, and less than 15 m, the sum of the two components, 10 m and 5 m)
9. b (vertically, it falls 1 meter)
10. a (horizontally, it doesn't accelerate)
11. b (for an object moving in a circle, the net force must be directed toward the center of the circle)
12. b (for an object moving in a circle, the net force must be directed toward the center of the circle)
13. c (use the radius of the path the object, in this case the moon, is following)

Section B

1. Assuming no air resistance, there is only one force acting on the stone: the gravitational force. That means the net force on the stone is mg downward. From Newton's second law, this means that $mg = ma$ so that $a = g$.

Since the object is released from rest, the initial velocity is zero. Using the “combined” relationship with the y direction being downward, and using vertical displacement equal to 10 m downward, vertical acceleration 9.8 m/s^2 downward and the vertical initial velocity zero, we have

$$\begin{aligned}\Delta y &= v_{y,i}\Delta t + \frac{1}{2}a_y(\Delta t)^2 \\ (10 \text{ m}) &= 0 + \frac{1}{2}(9.8 \text{ m/s}^2)(\Delta t)^2\end{aligned}$$

and solving for Δt I get 1.43 s.

2. (a) I'll skip the picture but the variables are as follows. Using x for horizontal and y for upward, the horizontal displacement (Δx) is 7.00 m and the vertical displacement (Δy) is -0.650 m . These are both specified in the problem. Since the nozzle is held horizontally, I infer that there is no vertical component to the initial velocity (i.e., $v_{y,i}$ is zero). Furthermore, assuming no air resistance, there is no horizontal acceleration (a_x is zero) and the vertical acceleration (a_y) is -9.8 m/s^2 (negative because I set the y direction as upward).
 - (b) The “combined” equation for horizontal displacement would be

$$\begin{aligned}\Delta x &= v_{x,i}\Delta t + \frac{1}{2}a_x(\Delta t)^2 \\ (7.00 \text{ m}) &= v_{x,i}\Delta t + \frac{1}{2}(0)(\Delta t)^2 \\ (7.00 \text{ m}) &= v_{x,i}\Delta t\end{aligned}$$

where I used only of the information I know about the *horizontal* motion (the horizontal displacement was given as 7.00 m and the acceleration has no horizontal component).

- (c) The “combined” equation for vertical displacement would be

$$\begin{aligned}\Delta y &= v_{y,i}\Delta t + \frac{1}{2}a_y(\Delta t)^2 \\ (-0.650 \text{ m}) &= (0)\Delta t + \frac{1}{2}(-9.8 \text{ m/s}^2)(\Delta t)^2 \\ (-0.650 \text{ m}) &= (-4.9 \text{ m/s}^2)(\Delta t)^2\end{aligned}$$

where I used only the information I know about the *vertical* motion (the vertical displacement was given as 0.650 m below the starting point, the initial velocity has no vertical component and the acceleration 9.8 m/s² downward because it is in free fall). Note that I have set positive as upward.

- (d) To determine the initial horizontal velocity, I need to first solve for the elapsed time. From the second expression, I get that the elapsed time is 0.364 s. I then plug that into the first expression to get an initial horizontal velocity of 19.2 m/s.
3. For the moon to move in uniform circular motion, it must have a speed equal to $2\pi r/T$. Converting the period into seconds (2.36×10^6 s), I get a speed of 1.02×10^3 m/s.

Furthermore, the moon must be undergoing an acceleration (changing directions) at a rate equal to $2\pi v/T$ and directed toward the center of the circle. Plugging in the speed and the period (converted to seconds), I get an acceleration of 0.0146 m/s² (toward the Earth).

That means there must be a force acting on the moon equal to 2.72×10^{-3} m/s² times the moon's mass. In this case, that force is being provided by the gravitational attraction between the earth and the moon. Using Newton's universal law of gravitation and setting the gravitational force equal to the moon's mass times the acceleration, we have

$$G \frac{m_{\text{earth}} m_{\text{moon}}}{r^2} = m_{\text{moon}} (2.72 \times 10^{-3} \text{ m/s}^2)$$

This can be rearranged to get that the Earth's mass is equal to $(2.72 \times 10^{-3} \text{ m/s}^2) r^2 / G$. Plugging in 3.88×10^8 m for the radius of the circle (i.e., radius of the moon's orbit) and 6.67×10^{-11} Nm²/kg² for the gravitational constant, I get a mass of 6.04×10^{24} kg. This is slightly more than the actual value because the center of the moon's orbit is not coincident with the center of the earth.

4. (a) The speed is equal to the circumference divided by how long it takes to go all the way around:

$$\begin{aligned} v &= (2\pi r)/T \\ &= 2\pi(6.37 \times 10^6 \text{ m})/(24 \times 3600 \text{ s}) \end{aligned}$$

which gives a speed of 463.2 m/s.

- (b) The acceleration of an object in circular motion is directed toward the center of the circle and has a magnitude equal to:

$$\begin{aligned} a_r &= (2\pi v)/T \\ &= 2\pi(463.2 \text{ m/s})/(24 \times 3600 \text{ s}) \end{aligned}$$

which gives an acceleration of 0.0337 m/s^2 toward the center of the Earth (center of circular path in this case).

- (c) From Newton's second law, the net force on the student must equal the product of the student's mass and acceleration:

$$\begin{aligned} F_{\text{net}} &= ma \\ &= (70 \text{ kg})(0.0337 \text{ m/s}^2) \end{aligned}$$

which gives a net force of 2.36 N (toward the center of the circle).

- (d) The gravitational force is equal to mg or 686 N . If that is equal to the net force (i.e., that is the only force acting) then there is no normal force at all. To find the velocity at which this would be true, we work backwards. From Newton's second law, the acceleration would have to be F_{net}/m or 9.8 m/s^2 . Knowing the acceleration, we can use $a = v^2/r$ to get the velocity (since r is known). Solve for the velocity to get

$$\begin{aligned} v &= \sqrt{ar} \\ &= \sqrt{(9.8 \text{ m/s}^2)(6.37 \times 10^6 \text{ m})} \end{aligned}$$

which gives a speed of $7.9 \times 10^3 \text{ m/s}$. This is the speed an object must travel (at the earth's surface) to be "in orbit" (i.e., without falling down to the ground).

5. The average acceleration is defined as $\Delta v/\Delta t$. So, we first need to find the change in velocity during the 10 seconds. This means we need to identify the velocity at the beginning and the velocity at the end. Since velocity is defined as $\Delta x/\Delta t$ and the graph plots position (x) vs. time (t), that means we can obtain an estimate of the velocity by looking at the slope. At the beginning, the slope is about 3 m/s (draw a straight line tangent to the curve at that point). At the end, the slope is about -3 m/s (the same as the beginning but negative). That means the change in velocity is about -6 m/s over the 10-second interval. Divide by 10 s to get an average acceleration of -0.6 m/s^2 .

6. (a) There are two forces acting on the box: the gravitational force pulling straight downward and a normal force pushing away from the inclined surface.

Since we know the net force must be parallel to the surface, we only need to calculate the forces parallel to the surface. Of the forces that are acting, only the gravitational force has a component parallel to the surface (the normal force is perpendicular to the surface).

The magnitude of the gravitational force is $F_g = mg = (9.8 \text{ N/kg})(10 \text{ kg}) = 98 \text{ N}$.

The component directed down the inclined surface is 59 N. You can get this a number of ways (e.g., using the cosine of $+53^\circ$ or -53° , or the sine of 37°). You know it isn't $\cos 37^\circ$ because the cosine gives you the component in the direction of 0° and the gravitational force is not 37° from "down the surface".

In any event, since the only force acting parallel to the surface is this component of the gravitational force, the net force must also be 59 N and be directed down the surface.

- (b) From Newton's second law, the acceleration must be the net force divided by the mass. Divide 59 N by 10 kg to get an acceleration of 5.9 m/s^2 down the surface.
- (c) Since the acceleration is constant (not varying), we can use the "combined" relationship to find the displacement:

$$\Delta x = v_{x,i}(\Delta t) + \frac{1}{2}a_x(\Delta t)^2.$$

Using \hat{x} directed up the surface and plugging in what we know,

$$\begin{aligned} \Delta x &= v_{x,i}(\Delta t) + \frac{1}{2}a_x(\Delta t)^2 \\ &= (7.0 \text{ m/s})(3.0 \text{ s}) + \frac{1}{2}(-5.9 \text{ m/s}^2)(3.0 \text{ s})^2 \\ &= (21.0 \text{ m}) - (26.55 \text{ m}) \end{aligned}$$

which gives a total displacement of (-5.55 m) up the incline or 5.55 m down from the original position. [note that the acceleration is *not* 7.0 m/s ! That is the velocity.]

- (d) If there is friction, then the acceleration will be different. To find the friction, you must first find the normal force, which you then

multiply by the coefficient (0.1 in this case). Once you find the friction, the process is the same as what we did in parts (a), (b), (c) and (d).

To find the normal force, apply Newton's second law to the direction perpendicular to the surface. Since the object doesn't accelerate in that direction, the net force in that direction must be zero. Since the friction force is parallel to the surface, not perpendicular, the only two forces we need to concern ourselves with is the normal force (pointing away from the surface) and a component of the gravitational force (pointing into the surface).

Since the component of F_g parallel to the surface was obtained using the cosine of 53° (or the sine of 37°), the component perpendicular to the surface is obtained using the sine of 53° (or the cosine of 37°). This gives a value of 78.3 N into the surface. That means that the normal force must be 78.3 N out of the surface.

Now we can find the friction force by multiplying that by 0.1 to get 7.83 N.

In this case, the friction force is directed down the incline, since the object is sliding up the incline. The total force parallel to the surface is now the sum of the friction force (7.83 N down the incline) and the component of the gravitational force parallel to the surface (59 N down the incline). That means the net force is 66.83 N down the incline.

Divide by the mass (10 kg) to get an acceleration of 6.683 m/s^2 down the surface.

Since we are given both the initial (7.0 m/s up the surface) and final velocities (zero), we can get the time by using the definition of average acceleration:

$$a_{\text{avg}} = \frac{\Delta v}{\Delta t}$$

and plugging in 6.683 m/s^2 (down) for the acceleration and 7.0 m/s (down) for the change in velocity (since it went from 7.0 m/s up to zero), we get a time of 1.047 s.

We also know the average velocity must be 3.5 m/s up the surface (half way between the initial and final velocities) and we can then

use the definition of average velocity

$$v_{\text{avg}} = \frac{\Delta x}{\Delta t}$$

to go get the displacement. I get 3.67 m up the incline.

Notice that at this point the box is stationary, so we would need to know the coefficient of static friction in order to predict what happens at that point (whether it stays stationary or starts to move down the surface). In addition, once it starts moving down the surface, the friction is no longer pointing downward but rather is pointing upward. That would change the net force and, consequently, the acceleration.