

Please email me if you have questions about the exam.

### Section A

1. e (the coefficient represents what fraction the friction is relative to the normal force)
2. b (the gravitational force is given by Newton's simplified law of gravitation)
3. c (tension force on box is in same direction as string)
4. c (the bullet fired downward has a non-zero vertical velocity whereas the bullet fired horizontally has a zero vertical velocity)
5. d (the horizontal velocity is the same as what it was on the table)
6. d (since there is no horizontal acceleration during free fall, there is no change in the horizontal velocity)
7. a (the vertical velocity is the same as what it was on the table, where it wasn't moving vertically)
8. d (it ends 10 m south of the initial position)
9. b (it must be greater than either 5, the eastward component, or 10 m, the southward component, and less than 15 m, the sum of the two components, 10 m and 5 m)
10. b (vertically, it falls 1 meter)
11. a (with no horizontal forces the ball will not accelerate horizontally)

### Section B

1. To find the net force, first draw a diagram (not shown here, but you should still draw one yourself) then choose a coordinate system. I'll choose  $\hat{x}$  and  $\hat{y}$  to be east and north, respectively.

Next, we determine the component values for each force, along the two component directions (east and north). There are lots of ways to do

this, as your choice of sine vs. cosine depends on what angle you use. However, you do it, these are the components I get:

$$\begin{aligned}
 F_{1,x} &= 9.397 \text{ N} \\
 F_{1,y} &= 3.420 \text{ N} \\
 F_{2,x} &= 10.000 \text{ N} \\
 F_{2,y} &= -17.320 \text{ N} \\
 F_{3,x} &= -22.981 \text{ N} \\
 F_{3,y} &= -19.284 \text{ N} \\
 F_{4,x} &= 0 \text{ N} \\
 F_{4,y} &= 49 \text{ N} \\
 F_{g,x} &= 0 \text{ N} \\
 F_{g,y} &= -49 \text{ N}
 \end{aligned}$$

Then, add up all the  $x$  components to get

$$F_{\text{net},x} = -3.584 \text{ N}$$

and add up all the  $y$  components to get

$$F_{\text{net},y} = -33.184 \text{ N}.$$

To get the magnitude, use

$$\begin{aligned}
 F_{\text{net}} &= \sqrt{F_{\text{net},x}^2 + F_{\text{net},y}^2} \\
 &= \sqrt{(-3.584 \text{ N})^2 + (-33.184 \text{ N})^2} \\
 &= 33.377 \text{ N}.
 \end{aligned}$$

To get the direction, use one of the inverse trigonometric functions. For example,

$$\begin{aligned}
 \theta &= \cos^{-1} \left( \frac{F_{\text{net},x}}{F_{\text{net}}} \right) \\
 &= \cos^{-1} \left( \frac{-3.584 \text{ N}}{33.377 \text{ N}} \right) \\
 &= 96.165^\circ.
 \end{aligned}$$

The actual direction must be opposite this, since both components are negative. Thus, the direction is  $264^\circ$  (or 6 degrees west of south).

2. (a) I'll skip the picture but the variables are as follows. Using  $x$  for horizontal and  $y$  for upward, the horizontal displacement ( $\Delta s_x$ ) is 7.00 m and the vertical displacement ( $\Delta s_y$ ) is  $-0.650$  m. These are both specified in the problem. Since the nozzle is held horizontally, I infer that there is no vertical component to the initial velocity (i.e.,  $v_{y,i}$  is zero). Furthermore, assuming no air resistance, there is no horizontal acceleration ( $a_x$  is zero) and the vertical acceleration ( $a_y$ ) is  $-9.8$  m/s<sup>2</sup> (negative because I set the  $y$  direction as upward).

- (b) The “combined” equation for horizontal displacement would be

$$\begin{aligned}\Delta s_x &= v_{x,i}\Delta t + \frac{1}{2}a_x(\Delta t)^2 \\ (7.00 \text{ m}) &= v_{x,i}\Delta t + \frac{1}{2}(0)(\Delta t)^2 \\ (7.00 \text{ m}) &= v_{x,i}\Delta t\end{aligned}$$

where I used only of the information I know about the *horizontal* motion (the horizontal displacement was given as 7.00 m and the acceleration has no horizontal component).

- (c) The “combined” equation for vertical displacement would be

$$\begin{aligned}\Delta s_y &= v_{y,i}\Delta t + \frac{1}{2}a_y(\Delta t)^2 \\ (-0.650 \text{ m}) &= (0)\Delta t + \frac{1}{2}(-9.8 \text{ m/s}^2)(\Delta t)^2 \\ (-0.650 \text{ m}) &= (-4.9 \text{ m/s}^2)(\Delta t)^2\end{aligned}$$

where I used only the information I know about the *vertical* motion (the vertical displacement was given as 0.650 m below the starting point, the initial velocity has no vertical component and the acceleration 9.8 m/s<sup>2</sup> downward because it is in free fall). Note that I have set positive as upward.

- (d) To determine the initial horizontal velocity, I need to first solve for the elapsed time. From the second expression, I get that the elapsed time is 0.364 s. I then plug that into the first expression to get an initial horizontal velocity of 19.2 m/s.

3. (a) There are two forces acting on the box: the gravitational force pulling straight downward and a normal force pushing away from the inclined surface.

Since we know the net force must be parallel to the surface, we only need to calculate the forces parallel to the surface. Of the forces that are acting, only the gravitational force has a component parallel to the surface (the normal force is perpendicular to the surface).

The magnitude of the gravitational force is  $F_g = mg = (9.8 \text{ N/kg})(10 \text{ kg}) = 98 \text{ N}$ .

The component directed down the inclined surface is 59 N. You can get this a number of ways (e.g., using the cosine of  $+53^\circ$  or  $-53^\circ$ , or the sine of  $37^\circ$ ). You know it isn't  $\cos 37^\circ$  because the cosine gives you the component in the direction of  $0^\circ$  and the gravitational force is not  $37^\circ$  from "down the surface".

In any event, since the only force acting parallel to the surface is this component of the gravitational force, the net force must also be 59 N and be directed down the surface.

- (b) From Newton's second law, the acceleration must be the net force divided by the mass. Divide 59 N by 10 kg to get an acceleration of  $5.9 \text{ m/s}^2$  down the surface.
- (c) Since the acceleration is constant (not varying), we can use the "combined" relationship to find the displacement:

$$\Delta s_x = v_{x,i}(\Delta t) + \frac{1}{2}a_x(\Delta t)^2.$$

Using  $\hat{x}$  directed up the surface and plugging in what we know,

$$\begin{aligned} \Delta s_x &= v_{x,i}(\Delta t) + \frac{1}{2}a_x(\Delta t)^2 \\ &= (7.0 \text{ m/s})(3.0 \text{ s}) + \frac{1}{2}(-5.9 \text{ m/s}^2)(3.0 \text{ s})^2 \\ &= (21.0 \text{ m}) - (26.55 \text{ m}) \end{aligned}$$

which gives a total displacement of  $(-5.55 \text{ m})$  up the incline or  $5.55 \text{ m}$  down from the original position. [note that the acceleration is *not*  $7.0 \text{ m/s}$ ! That is the initial velocity.]

- (d) If there is friction, then the acceleration will be different. To find the friction, you must first find the normal force, which you then multiply by the coefficient (0.1 in this case). Once you find the friction, the process is the same as what we did in parts (a), (b) and (c).

To find the normal force, apply Newton's second law to the direction perpendicular to the surface. Since the object doesn't accelerate in that direction, the net force in that direction must be zero. Since the friction force is parallel to the surface, not perpendicular, the only two forces we need to concern ourselves with is the normal force (pointing away from the surface) and a component of the gravitational force (pointing into the surface).

Since the component of  $F_g$  parallel to the surface was obtained using the cosine of  $53^\circ$  (or the sine of  $37^\circ$ ), the component perpendicular to the surface is obtained using the sine of  $53^\circ$  (or the cosine of  $37^\circ$ ). This gives a value of 78.3 N into the surface. That means that the normal force must be 78.3 N out of the surface.

Now we can find the friction force by multiplying that by 0.1 to get 7.83 N.

In this case, the friction force is directed down the incline, since the object is sliding up the incline. The total force parallel to the surface is now the sum of the friction force (7.83 N down the incline) and the component of the gravitational force parallel to the surface (59 N down the incline). That means the net force is 66.83 N down the incline.

Divide by the mass (10 kg) to get an acceleration of  $6.683 \text{ m/s}^2$  down the surface.

Now that we have the acceleration, we can use the combined equation to get the displacement (see part c). Alternatively, you can skip the combined equation and instead use the three separate equations, one at a time. For comparison, this is how you'd use the three equations.

Since we are given both the initial (7.0 m/s up the surface) and final velocities (zero), we can get the time by using the definition of average acceleration:

$$a_{\text{avg}} = \frac{\Delta v}{\Delta t}$$

and plugging in  $6.683 \text{ m/s}^2$  (down) for the acceleration and 7.0 m/s (down) for the change in velocity (since it went from 7.0 m/s up to zero), we get a time of 1.047 s.

We also know the average velocity must be 3.5 m/s up the surface (half way between the initial and final velocities) and we can then use the definition of average velocity

$$v_{\text{avg}} = \frac{\Delta x}{\Delta t}$$

to go get the displacement. I get 3.67 m up the incline.

Notice that at this point the box is stationary, so we would need to know the coefficient of static friction in order to predict what happens at that point (whether it stays stationary or starts to move down the surface). In addition, once it starts moving down the surface, the friction is no longer pointing downward but rather is pointing upward. That would change the net force and, consequently, the acceleration.